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TRANNOZ: A COMPUTER PROGRAM FOR ANALYSIS OF TRANSONIC THROAT FL--ETC(U)

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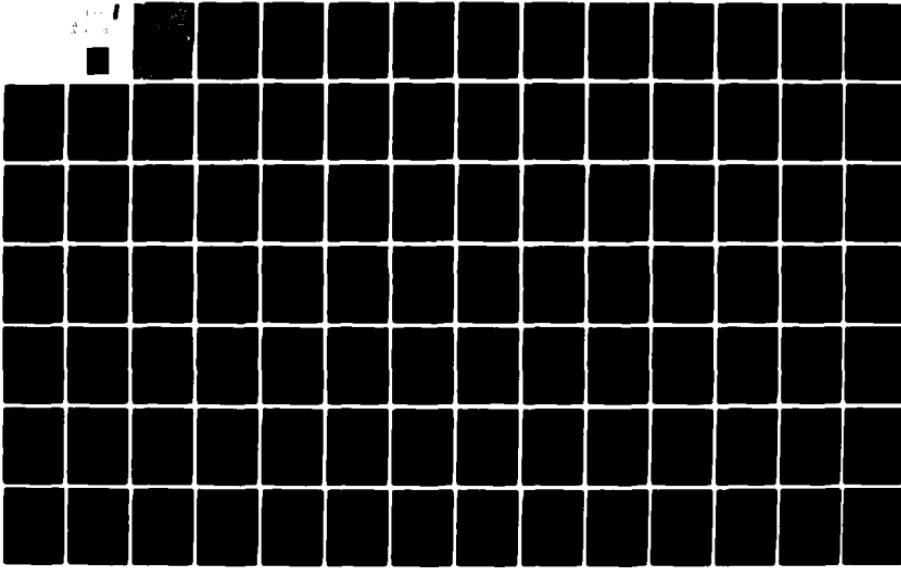
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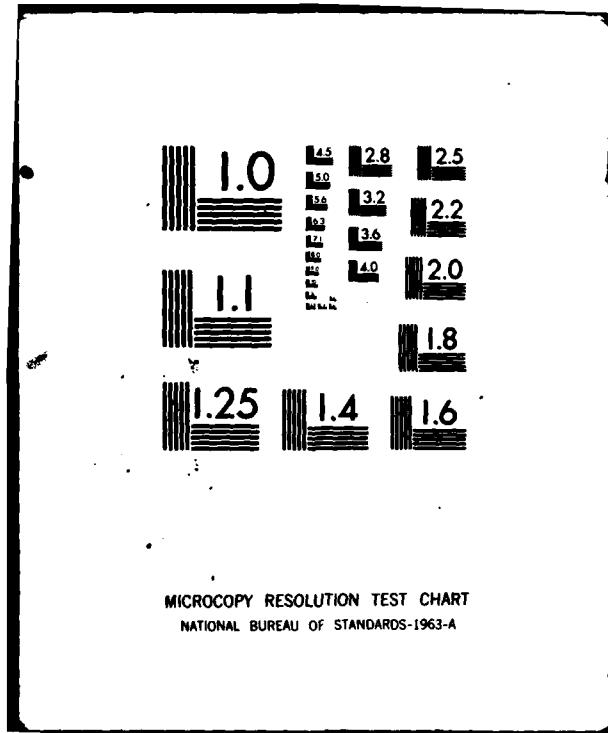
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# TRANNOZ: A COMPUTER PROGRAM FOR ANALYSIS OF TRANSONIC THROAT FLOW IN AXISYMMETRIC, PLANAR, AND ANNULAR SUPERSONIC NOZZLES

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by  
J. C. DUTTON  
A. L. ADDY



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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A FORTRAN computer program has been developed for the analysis of transonic throat flowfields in annular, planar, and axisymmetric supersonic nozzles. The program evaluates the series expansion solution developed by Dutton and Addy. The subroutines in the code are described together with the definitions of the input and output variables, detailed input instructions, and an example input file with the corresponding output. A brief summary of the theory upon which the expansion solution is based is also included.			

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TRANSONIC THROAT FLOW IN AXISYMMETRIC, PLANAR,  
AND ANNULAR SUPERSONIC NOZZLES

by  
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## NOMENCLATURE

A complete listing of the subroutine names and their functions, the input NAMELISTS, and the input and output variables is contained in Sections III.A and III.B. Other quantities not defined there are listed below:

<u>Text</u>	<u>Computer Program</u>	<u>Meaning</u>
a		speed of sound
a*		critical speed of sound
A*	ASTAR	throat area
C <sub>D</sub>	CD	discharge coefficient, defined in Eq. (II-32)
C <sub>D1</sub> ,C <sub>D2</sub> ,C <sub>D3</sub>	CD1,CD2,CD3	discharge coefficient constants, Eq. (II-33)
d	D	distance between the throat locations at the inner and outer walls, Fig. II.1
g,h		equations of the inner and outer nozzle wall contours in the x-y coordinate system, Fig. II.1
g <sub>1</sub> ,h <sub>1</sub> ,g <sub>2</sub> ,h <sub>2</sub>	G1,H1,G2,H2	dimensionless quantities defined in Eq. (II-25)
G,H		equations of the inner and outer nozzle wall contours in the R-Z coordinate system, Fig. II.1
M	M	Mach number
M*	MSTAR	ratio of the speed at a point to the critical speed of sound
O		used to denote physical order of magnitude
p/p <sub>0</sub>	PP0	static-to-stagnation pressure ratio
R	R	radial coordinate in cylindrical coordinate system, Fig. II.1

<u>Text</u>	<u>Computer Program</u>	<u>Meaning</u>
$\bar{R}_e$		average radius of curvature for the two bounding walls at the annular nozzle throat
$R_{e1}, R_{eo}$	RCI, RCS	radii of curvature of the inner and outer nozzle walls at the nozzle throat
$R_i, R_o$	RI, RO	radial coordinates of the inner and outer wall throat locations, Fig. II.1
$Re_{2d}$		Reynolds number based on sonic conditions and twice the throat separation distance, d
$u, v$	U, V	dimensionless velocity components in the x-y coordinate system defined in Eqs. (II-5) and (II-6), Fig. II.1
$\tilde{u}, \tilde{v}$		transonic perturbation velocity components defined in Eqs. (II-10) and (II-11)
$u_1, v_1, u_2, v_2,$ $u_3, v_3$	U1, V1, U2, V2, U3, V3	transonic perturbation velocity components of the first three orders defined by the expansions in Eqs. (II-19) and (II-20)
$U, V$		velocity components in the cylindrical R-Z coordinate system, Fig. II.1
$x, y$	X, Y	rotated coordinate system non-dimensionalized with respect to the throat separation distance, d, and oriented such that the y-axis lies along the minimum area cross-section and the origin is on the Z-axis of symmetry, Eqs. (II-3) and (II-4) and Fig. II.1
$y_i, y_o$	YI, YO	y-coordinates of the inner and outer throat wall locations, Fig. II.1

<u>Text</u>	<u>Computer Program</u>	<u>Meaning</u>
$z$	$z$	stretched axial coordinate defined in Eq. (II-25)
$z$	$z$	axial coordinate in the cylindrical coordinate system, Fig. II.1
$z^*$	ZSTAR	displacement of the x-y origin from the R-Z origin along the Z-axis of symmetry, Fig. II.1
$z_i, z_o$	ZI,ZO	axial coordinates of the inner and outer throat wall locations, Fig. II.1

Greek Symbols

$\beta$	BETA	inclination angle of the x-axis from the Z-axis of symmetry, positive counterclockwise, Fig. II.1
$\beta_1$	BETA1	dimensionless quantity defined in Eq. (II-25)
$\gamma$	G	specific heat ratio of the gas
$\epsilon$	EPS	expansion variable defined in Eq. (II-18)
$\eta$	ETA	parameter in the expansion variable definition, Eq. (II-18)
$\theta$	THETA	angle of inclination of the velocity vector from the x-axis, positive counterclockwise
$\rho$		density
$\rho^*$		critical density

## I. INTRODUCTION

Annular supersonic nozzles constitute an integral part of a number of devices of practical importance. These applications include turbofan bypass nozzles, unconventional propulsion nozzles such as the spike, plug, and expansion-deflection designs, as well as the supersonic-supersonic ejector and axial-flow aerodynamic window. In order to analyze the supersonic flowfield in these nozzles using either the method-of-characteristics or a finite difference technique, an accurate initial value line is required. One natural place to start these calculations is in the throat region of the nozzles using an appropriate analysis of the transonic flowfield which occurs there. Given this starting line, the marching-type computations for the steady, supersonic portion of the flowfield can then proceed in the streamwise direction.

Several methods have been utilized to analyze transonic flow in the throat region of annular supersonic nozzles. These methods include inverse techniques [1,2]<sup>†</sup>, series expansion methods [3-5], time-dependent numerical techniques [6-8], the method of integral relations [9], and type-dependent numerical relaxation [10]. However, each of these previous methods either applies only to a specialized class of annular

---

<sup>†</sup>Numbers in brackets refer to entries in REFERENCES.

nozzles, such as those with cylindrical centerbodies or located a large distance from the axis of symmetry, or the numerical technique employed requires an inordinate amount of computer time and/or memory. In addition, the inverse techniques require iteration for the direct problem of analyzing the flowfield in a nozzle of given contour. Only the recent numerical methods of Cline [8] and Brown, et al. [10] have shown promise of analyzing nozzle throat flows with reasonable amounts of computer time.

What is desired, therefore, is a direct method which can accurately and economically describe the transonic flowfield in the throat region of a large class of annular, supersonic nozzles. Just such a method was developed by the present authors in [11]. A series expansion technique similar to the one used originally by Hall [12] for axisymmetric and planar nozzles was utilized to find an approximate solution to the inviscid, irrotational governing equations. This solution may be applied to a variety of annular nozzle configurations including those for which the centerbody and outer wall contours are both circular arcs or, alternately, those for which one boundary is straight. In addition, the main flow direction may be either parallel or inclined with respect to the axis of symmetry. In the limit as the centerbody radius approaches zero and the outer wall, respectively, the solutions for the simpler cases of axisymmetric and planar nozzles are

obtained so that these cases may also be analyzed. Since the solution is of the series expansion type, one of its major advantages is the speed and reliability of its numerical implementation, making feasible parametric studies and iterative calculations.

The purpose of the present report is to describe the FORTRAN computer program TRANNOZ which has been developed to implement the solution just discussed. After a brief summary of the theoretical development of the expansion solution, the computer code is discussed in detail including descriptions of its subroutines and functions and of its input and output variables. Input instructions are also given together with a sample input file and the resulting output. A listing of the program is included in the Appendix.

## II. THEORY

A sketch of the configuration to be analyzed is shown in Fig. II.1. It consists of an annular supersonic nozzle which, in general, may be inclined with respect to the axis of symmetry. The R-Z coordinate system is the standard cylindrical coordinate system, while the x-y coordinate system is rotated in such a manner that the y-axis lies along the cross section of minimum area in the nozzle throat. The x-axis is perpendicular to the y-axis, and the origin of this coordinate system is located on the Z-axis of symmetry a distance  $Z^*$  from the R-Z origin. The angle  $\beta$  is the inclination angle between the x-axis and the Z-axis, and  $d$  is the distance in the R-Z coordinate system between the inner and outer throat wall locations. The coordinates of these last two points as well as those of the x-y origin and the equations of the inner and outer wall contours in the meridional plane are also given in both the R-Z and x-y coordinate systems in the figure. It is to be noted that for the general case of an inclined, annular nozzle the minimum area cross section does not correspond to the cross section of minimum distance between the nozzle walls. Because of the radial factor involved in calculating the annular area, the minimum area cross section is located nearer the axis of symmetry than the minimum distance cross section. More will be said about this when subroutine ARMIN is discussed.

Under the assumptions of steady, inviscid, irrotational, adiabatic flow of a perfect gas, the governing equations in the cylindrical coordinate system can be taken as the irrotationality condition and the "gas dynamic equation" [13],

$$U_R - V_Z = 0 \quad (\text{III-1})$$

$$(U^2 - a^2) U_Z + (V^2 - a^2) V_R + 2UVU_R - \frac{a^2 V}{R} = 0 \quad (\text{III-2})$$

$a \equiv$  speed of sound ,

where the subscripts are used to denote partial differentiation with respect to  $Z$  and  $R$ . Transforming from the  $R-Z$  to the rotated  $x-y$  coordinate system where lengths are non-dimensionalized with respect to the throat separation distance,  $d$ , and velocities with respect to the critical speed of sound,  $a^*$ ,

$$x = \frac{(Z-Z^*)}{d} \cos\beta + \frac{R}{d} \sin\beta \quad (\text{III-3})$$

$$y = - \frac{(Z-Z^*)}{d} \sin\beta + \frac{R}{d} \cos\beta \quad (\text{III-4})$$

$$u = \frac{U}{a^*} \cos\beta + \frac{V}{a^*} \sin\beta \quad (\text{III-5})$$

$$v = - \frac{U}{a^*} \sin\beta + \frac{V}{a^*} \cos\beta , \quad (\text{III-6})$$

and using the adiabatic relation,

$$\left(\frac{a}{a^*}\right)^2 = \frac{\gamma+1}{2} - \frac{\gamma-1}{2} (u^2 + v^2) , \quad (\text{III-7})$$

the governing equations become

$$u_y - v_x = 0 \quad (\text{III-8})$$

$$\begin{aligned} \left(1-u^2 - \frac{\gamma-1}{\gamma+1} v^2\right) u_x - \frac{4}{\gamma+1} uv u_y + \left(1-v^2 - \frac{\gamma-1}{\gamma+1} u^2\right) v_y \\ + \frac{\left(1 - \frac{\gamma-1}{\gamma+1} u^2 - \frac{\gamma-1}{\gamma+1} v^2\right) (v \cos\beta + u \sin\beta)}{y \cos\beta + x \sin\beta} = 0. \end{aligned} \quad (\text{II-9})$$

The next step in the analytical development involves introduction of the transonic perturbation velocity components,  $\tilde{u}$  and  $\tilde{v}$ , by the relations

$$u = 1 + \tilde{u} \quad (\text{II-10})$$

$$v = \tilde{v}, \quad (\text{II-11})$$

which, when substituted into Eqs. (II-8) and (II-9) result in

$$\tilde{u}_y - \tilde{v}_x = 0 \quad (\text{II-12})$$

$$\begin{aligned} \left(-2\tilde{u}-\tilde{u}^2 - \frac{\gamma-1}{\gamma+1} \tilde{v}^2\right) \tilde{u}_x - \frac{4}{\gamma+1} (1+\tilde{u}) \tilde{v} \tilde{u}_y + \left(\frac{2}{\gamma+1} - \tilde{v}^2 - 2 \frac{\gamma-1}{\gamma+1} \tilde{u} - \frac{\gamma-1}{\gamma+1} \tilde{u}^2\right) \tilde{v}_y \\ + \frac{\left(\frac{2}{\gamma+1} - 2 \frac{\gamma-1}{\gamma+1} \tilde{u} - \frac{\gamma-1}{\gamma+1} \tilde{u}^2 - \frac{\gamma-1}{\gamma+1} \tilde{v}^2\right) [\tilde{v} \cos\beta + (1+\tilde{u}) \sin\beta]}{y \cos\beta + x \sin\beta} = 0. \end{aligned} \quad (\text{II-13})$$

The boundary conditions for this inviscid analysis are that the nozzle walls must be streamlines. Taking  $y=g(x)$  and  $y=h(x)$  as the equations for the inner and outer wall contours, respectively, the boundary conditions may be written as

$$\tilde{v}(x, g(x)) = [1+\tilde{u}(x, g(x))] g'(x) \quad (\text{II-14})$$

$$\tilde{v}(x, h(x)) = [1+\tilde{u}(x, h(x))] h'(x), \quad (\text{II-15})$$

where the prime is used to denote differentiation with respect to  $x$ .

To this point in the development no approximations to either the governing partial differential equations or the

boundary conditions have been made. In order to proceed, an expansion parameter must be defined so that the perturbation velocity components can be expanded in appropriate series and substituted into the equations and boundary conditions.

Based on the experience of Kliegel and Levine [14] and Thompson and Flack [5], the expansion parameter used in this investigation is

$$\epsilon = (\bar{R}_e + \eta)^{-1} \quad (\text{II-16})$$

where  $\bar{R}_e$  is an average dimensionless radius of curvature for the two bounding walls. The parameter  $\eta$  is included in order to improve the convergence properties of the series solution for nozzles with a small wall radius of curvature. For  $\eta > 1$ ,  $\epsilon$  is less than unity regardless of how small  $\bar{R}_e$  may be. Defining  $\bar{R}_e$  in terms of the second derivatives of the equations for the wall contours,

$$\bar{R}_e = \frac{2}{h''(0) - g''(0)} \quad (\text{II-17})$$

the definition of  $\epsilon$  becomes,

$$\epsilon = \frac{h''(0) - g''(0)}{2 + \eta[h''(0) - g''(0)]} \quad (\text{II-18})$$

This is the definition actually used in the evaluation of the series solution.

The solution technique then proceeds by investigating the orders of magnitude of the various terms in the equations and boundary conditions and by defining appropriate  $O(1)$  quantities.

Expanding the perturbation velocity components  $\tilde{u}$  and  $\tilde{v}$  as,

$$\tilde{u}(z,y) = u_1(z,y)\epsilon + u_2(z,y)\epsilon^2 + u_3(z,y)\epsilon^3 + \dots \quad (\text{II-19})$$

$$\tilde{v}(z,y) = \left[ \frac{\gamma+1}{2} \epsilon \right]^{1/2} \left[ v_1(z,y)\epsilon + v_2(z,y)\epsilon^2 + v_3(z,y)\epsilon^3 + \dots \right], \quad (\text{II-20})$$

substituting into governing Eqs. (II-12) and (II-13) and boundary conditions (II-14) and (II-15), and gathering coefficients of like powers of  $\epsilon$  results in the formulations for the various solution orders in the expansion technique. For the first order solution, the governing equations are

$$\frac{\partial u_1}{\partial y} - \frac{\partial v_1}{\partial z} = 0 \quad (\text{II-21})$$

$$- 2u_1 \frac{\partial u_1}{\partial z} + \frac{\partial v_1}{\partial y} + \frac{\beta_1 + v_1}{y} = 0 \quad (\text{II-22})$$

with corresponding boundary conditions

$$v_1(z, y_i) = g_1 + g_2 z \quad (\text{II-23})$$

$$v_1(z, y_o) = h_1 + h_2 z. \quad (\text{II-24})$$

where,

$$\begin{aligned} z &\equiv \left[ \frac{\gamma+1}{2} \epsilon \right]^{-1/2} x & \beta_1 &\equiv \left[ \frac{\gamma+1}{2} \right]^{-1/2} \epsilon^{-3/2} \tan\beta \\ g_1 &\equiv \left[ \frac{\gamma+1}{2} \right]^{-1/2} \epsilon^{-3/2} g'(0) & h_1 &\equiv \left[ \frac{\gamma+1}{2} \right]^{-1/2} \epsilon^{-3/2} h'(0) \\ g_2 &\equiv \frac{2g''(0)}{h''(0) - g''(0)} & h_2 &\equiv \frac{2h''(0)}{h''(0) - g''(0)}. \end{aligned} \quad (\text{II-25})$$

In the definitions in (II-25) note, in particular, the transformation from the coordinate  $x$  to the stretched, axial

coordinate z. The formulations for the higher order solutions are similar although they contain many more terms than the first order formulation.

Equations (II-21)-(II-24) and the corresponding equations for the higher order solutions are the ones which must be solved in order to obtain the  $(u_1, v_1)$ ,  $(u_2, v_2)$ ,  $(u_3, v_3)$ , ... perturbation velocity components. As discussed in [11], the series solution has been carried to the third order using the method of Hall [12]. The result is an approximate, analytical solution for the perturbation velocities consisting of a rather large and algebraically complicated set of constants and functions. However, because of the closed-form nature of the solution, these quantities can be rapidly evaluated in a straightforward manner using a digital computer.

With the resulting expressions for the  $(u_1, v_1)$ ,  $(u_2, v_2)$ , and  $(u_3, v_3)$  components determined, other quantities of interest may also be found. The series expansions for the following flowfield variables are given below: the velocity components u and v in the x-y coordinate system;  $M^*$ , the ratio of the local speed to the critical speed of sound;  $\theta$ , the angle of inclination of the velocity vector from the x-axis; the Mach number, M; and the local static-to-stagnation pressure ratio,  $p/p_0$ :

$$u(z,y) = 1 + \tilde{u} = 1 + u_1 \epsilon + u_2 \epsilon^2 + u_3 \epsilon^3 + \dots \quad (\text{II-26})$$

$$v(z, y) = \tilde{v} = \left[ \frac{\gamma+1}{2} \epsilon \right]^{1/2} [v_1 \epsilon + v_2 \epsilon^2 + v_3 \epsilon^3 + \dots] \quad (\text{II-27})$$

$$M^*(z, y) = (u^2 + v^2)^{1/2} = 1 + u_1 \epsilon + u_2 \epsilon^2 + \left[ u_3 + \frac{\gamma+1}{4} v_1^2 \right] \epsilon^3 + \dots \quad (\text{II-28})$$

$$\theta(z, y) = \tan^{-1}(v/u) = \left[ \frac{\gamma+1}{2} \epsilon \right]^{1/2} [v_1 \epsilon + (v_2 - u_1 v_1) \epsilon^2 + (v_3 - u_1 v_2 - u_2 v_1 + u_1^2 v_1) \epsilon^3 + \dots] \quad (\text{II-29})$$

$$M(z, y) = \left[ \frac{\frac{2}{\gamma+1} M^*^2}{1 - \frac{\gamma-1}{\gamma+1} M^*^2} \right]^{1/2} = 1 + \left( \frac{\gamma+1}{2} \right) [u_1 \epsilon + [u_2 + \frac{3}{4} (\gamma-1) u_1^2] \epsilon^2 + [u_3 + \frac{\gamma+1}{4} v_1^2 + \frac{3}{2} (\gamma-1) u_1 u_2 + \frac{(5\gamma^2 - 8\gamma + 3)}{8} u_1^3] \epsilon^3 + \dots] \quad (\text{II-30})$$

$$\frac{p}{p_0} (z, y) = \left[ 1 - \frac{\gamma-1}{\gamma+1} M^*^2 \right]^{\gamma/(\gamma-1)} = \left( \frac{2}{\gamma+1} \right)^{\gamma/(\gamma-1)} \left[ 1 - \gamma [u_1 \epsilon + u_2 \epsilon^2 + (u_3 + \frac{\gamma+1}{4} v_1^2 - \frac{\gamma+1}{6} u_1^3) \epsilon^3 + \dots] \right]. \quad (\text{II-31})$$

Another quantity of interest is the discharge or flow coefficient,  $C_D$ , which is defined as the ratio of the actual nozzle mass flowrate to that obtained from the ideal approximation of uniform, sonic flow at the throat,

$$C_D = \int_{y_1}^{y_0} \left[ \frac{p}{\rho^*} u \frac{dA}{A^*} \right]_{x=0} . \quad (\text{II-32})$$

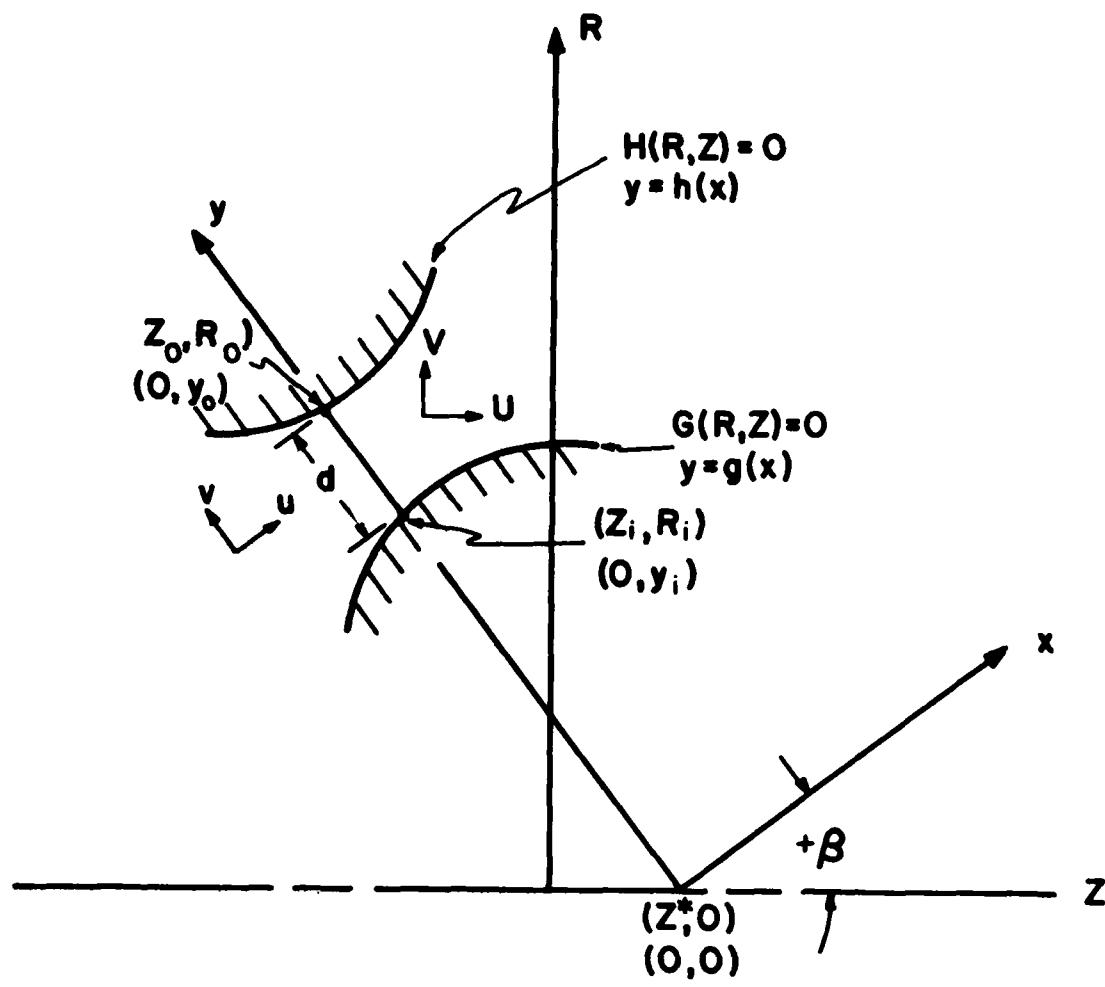
Substituting the appropriate expressions for the quantities in the integrand and carrying out the integration, the relation

for  $C_D$  becomes

$$C_D = 1 - \frac{(\gamma+1)\epsilon^2}{\left(\frac{y_e^2-y_i^2}{y_e^2}\right)} [C_{D1} + C_{D2}\epsilon + C_{D3}\epsilon^2 + \dots] \quad (\text{II-33})$$

where  $C_{D1}$ ,  $C_{D2}$ , and  $C_{D3}$  are constants.

Thus with the  $(u_1, v_1)$ ,  $(u_2, v_2)$ ,  $(u_3, v_3)$  transonic perturbation velocity components and the  $C_{D1}$ ,  $C_{D2}$ ,  $C_{D3}$  discharge coefficient constants determined, all of the flowfield variables of interest are known to third order in the present series approximations. For further details concerning the development of the expansion solution including the extensive series of checks and parametric studies which have been performed, reference [11] should be consulted.



**Figure II.1 Configuration for throat flowfield analysis of annular supersonic nozzles**

### III. COMPUTER PROGRAM TRANNOZ

The TRANNOZ computer code is a FORTRAN program for analyzing the transonic flowfield in the throat region of annular supersonic nozzles. This task is accomplished by evaluating the series expansion solution developed in [11] and outlined in the preceding chapter. Since the series evaluation consists essentially of the straightforward calculation of a set of constants and functions, and since the solution can be applied to a variety of nozzle configurations including the annular, planar, and axisymmetric cases, many nozzle throat flowfields of interest can be calculated in a routine and inexpensive manner. This feature makes possible parametric studies and iterative calculations as might be necessary, for example, in a design situation.

#### A. PROGRAM, FUNCTIONS, AND SUBROUTINES

The TRANNOZ code consists of a main program, two function subprograms, and twelve subroutines. A brief description of each of these routines is given below. A complete listing of TRANNOZ is contained in the Appendix.

##### 1. PROGRAM MAIN

The main program reads and writes the input variables, calls subroutines ARMIN and DISCO to calculate some initial parameters necessary in the series evaluation, calls the four main worker subroutines CONTOUR, STLINE, XPLANE, and ZPLANE as

TRANNOZ PAGE BLANK OUT FILE

desired, and writes the results. It is to be emphasized that under normal circumstances all reading and writing is done by program MAIN. If an error condition is encountered, a limited number of diagnostics are written by subroutine ERROR (see below).

## 2. FUNCTION IBND

Function IBND is the equation of the inner wall contour in the explicit form  $R=IBND(Z)$ . In the present form of IBND, the inner boundary may be either a circular arc or a straight line in the meridional plane, although any contour which satisfies the assumptions of the analysis could be used.

## 3. FUNCTION OBND

Function OBND is the equation of the outer boundary in the explicit form  $R=OBND(Z)$ . As for the inner boundary, TRANNOZ allows the outer wall contour to be either a circular arc or a straight line in the meridian plane, although other contours could also be utilized.

## 4. SUBROUTINE ARMIN

Subroutine ARMIN locates the cross section of minimum area in the nozzle throat and calculates some initial parameters necessary in evaluating the series solution. As discussed briefly in the previous chapter, for the general case of an inclined, annular nozzle the minimum area cross section does not coincide with the cross section of minimum distance between the nozzle walls, so that the throat location is not

known *a priori*. ARMIN employs an iterative, numerical technique [5] to locate the throat whereby the location of the outer wall point is first fixed and the location of the inner wall point is varied until a minimum in the cross-sectional area is found. The inner wall point is then fixed and the outer boundary point location is varied until the area is again minimized. This process is continued until the fractional change in these successively determined minimum areas is less than  $10^{-10}$ . This technique has been thoroughly tested by substituting the throat wall locations found numerically into the algebraic equations resulting from the constrained minimization problem<sup>†</sup> of locating the throat of an annular nozzle with circular arc contours. In all cases, the equations were found to be satisfied to within a high degree of numerical accuracy thus verifying the method.

With the throat location determined, ARMIN then evaluates the following geometrical parameters shown in Fig. II.1:  $A^*$ ,  $d$ ,  $z^*$ ,  $\beta$ ,  $y_i$ ,  $y_o$ ,  $g'(0)$ ,  $h'(0)$ ,  $g''(0)$ , and  $h''(0)$  where  $A^*$  is the throat area and  $y=g(x)$  and  $y=h(x)$  are the equations of the inner and outer contours in the x-y coordinate system. The dimensionless parameters,  $\epsilon$ ,  $g_1$ ,  $h_1$ ,  $g_2$ ,  $h_2$ , and  $\beta_1$ , defined in Eqs. (II-18) and (II-25), are also calculated.

---

<sup>†</sup>After elimination of the two Lagrange multipliers, the formulation of the constrained minimization problem consists of four nonlinear, simultaneous, algebraic equations for the coordinates of the inner and outer throat wall points  $(z_i, R_i)$ ,  $(z_o, R_o)$  in terms of the coordinates of the centers and radii of curvature of the circular arc boundaries.

### 5. SUBROUTINE DISCO

Subroutine DISCO evaluates the discharge coefficient,  $C_D$ , for the nozzle under consideration by calling subroutine AATTRANS to calculate the constants  $C_{D1}$ ,  $C_{D2}$ , and  $C_{D3}$  in the series approximation for  $C_D$ , Eq. (II-33).

The next four subroutines are called at the user's discretion to perform the major functions of the program.

### 6. SUBROUTINE CONTOUR

Subroutine CONTOUR finds the R-Z coordinates of the points on contours of constant Mach number, M, dimensionless speed ratio,  $M^*$ , or static-to-stagnation pressure ratio,  $p/p_0$ . A maximum of 53 points is allowed on each contour. A contour point is found on both the inner and outer boundaries and the remaining points are equally spaced in the y-coordinate from  $y_i$  to  $y_o$ . If the contour points at  $y=y_i$  and  $y=y_o$  are essentially coincident with the adjacent points on the boundaries (difference in y coordinates less than  $10^{-3}$ ), they are omitted. The order of the points on output is from the outer wall to the inner and all of the flow quantities along the contour are printed.

### 7. SUBROUTINE STLINE

Subroutine STLINE calculates the flowfield variables along a supersonic initial value line for starting method-of-characteristics or finite difference computations for the supersonic portion of the flowfield downstream from the nozzle

throat. The starting line which is calculated is the constant Mach number contour from the throat wall location with the higher Mach number. This line is employed since, under the assumptions of the analysis, it is the most accurate of the alternatives considered. It is realized that this initial value line may not be compatible with the particular algorithm used to analyze the supersonic flowfield. However, routines to evaluate other starting lines can easily be developed by using STLINE as an example.

#### 8. SUBROUTINE XPLANE

Subroutine XPLANE evaluates the flowfield variables of interest at a specified number of points along planes of constant x-coordinate (Fig. II.1) in the nozzle throat. The points are equally spaced in the y-coordinate from  $y_i$  to  $y_o$ , and a maximum of 51 points on each plane is allowed. This subroutine has been used primarily to test the series solution obtained in [11] for the general annular configuration against previous solutions for simpler geometries.

#### 9. SUBROUTINE ZPLANE

Subroutine ZPLANE computes the flowfield quantities at a specified number of points along planes of constant axial coordinate, Z. A maximum of 51 points is allowed on each plane, and they are equally spaced in the radial coordinate, R, from the inner to the outer wall.

The next six subroutines are secondary routines and are called a number of times by the main worker subroutines in order to carry out specific tasks.

#### 10. SUBROUTINE TRRZXY

Subroutine TRRZXY transforms the coordinates of a point from the R-Z cylindrical coordinate system to the dimensionless x-y system. This transformation is the one indicated by Eqs. (II-3) and (II-4).

#### 11. SUBROUTINE TRXYRZ

Subroutine TRXYRZ carries out the coordinate transformation of a point from the rotated x-y coordinate system to the R-Z cylindrical system. This is the inverse of the transformation expressed in Eqs. (II-3) and (II-4).

#### 12. SUBROUTINE ITER

Subroutine ITER is a general iteration subroutine used to find the value of the independent variable corresponding to a given value of the dependent variable in a functional relationship. The iterations are continued until error tests on either the independent or dependent variable are satisfied. This routine is used by subroutine CONTOUR for finding points along contours of constant  $M$ ,  $M^*$ , or  $p/p_0$ .

#### 13. SUBROUTINE VARSOR

Subroutine VARSOR is utilized to determine which of the three dependent variables  $M$ ,  $M^*$ , or  $p/p_0$  is being held constant

along a contour. It is used in conjunction with the iteration subroutine ITER by worker routine CONTOUR.

#### 14. SUBROUTINE ERROR

Subroutine ERROR writes a limited number of diagnostics for error conditions encountered in other subroutines and terminates program execution. Four of the diagnostics are involved with the iterations in subroutine ARMIN for locating the minimum area section in the nozzle throat and initializing the solution parameters. Another diagnostic is written when the iterations in subroutine CONTOUR for a given point on a contour do not converge. The final diagnostic is written when subroutine STLINE is called and the Mach number at neither of the throat wall locations is supersonic. Since none of these error conditions has ever been encountered, they will not be discussed in further detail.

#### 15. SUBROUTINE AATRANS

Subroutine AATRANS is the longest routine in the program. It evaluates all of the constants, functions, and flowfield quantities in the three term series expansion solution developed in [11]. In the form presented here, all of the constants and functions in AATRANS are double precision variables so that annular nozzles in the planar limit as the dimensionless distance from the axis of symmetry becomes very large, can also be analyzed without incurring significant roundoff errors. The penalty for adding this feature is

increased compilation and execution times, but since the program is extremely fast the effect of this penalty is minimal.

#### B. INPUT AND OUTPUT VARIABLES

As will be discussed in more detail in the next section, input to the TRANNOZ code is achieved through the use of six NAMELISTS: PARAM, CONTROL, NAMECON, NAMEST, NAMEXPL, and NAMEZPL. A description of each of the input variables which comprise these NAMELISTS follows.

##### 1. NAMELIST PARAM

NGEOM----an integer variable describing the nozzle geometry, Fig. III.1. If NGEOM=1, both the inner and outer boundaries are circular arcs in the meridional plane. If NGEOM=2, the inner boundary is a straight line and the outer boundary is a circular arc. If NGEOM=3, the inner boundary is a circular arc and the outer boundary is a straight line. (default=1)

AI-----Z coordinate of the center of curvature of the inner boundary if it is a circular arc (NGEOM=1 or 3) or slope of the inner boundary if it is a straight line (NGEOM=2), Fig. III.1.

BI-----R coordinate of the center of curvature of the inner boundary if it is a circular arc (NGEOM=1 or 3) or intercept of the inner boundary if it is a straight line (NGEOM=2), Fig. III.1.

RCI-----radius of curvature of the inner boundary if it is a circular arc (NGEOM=1 or 3), Fig. III.1.

A0-----Z coordinate of the center of curvature of the outer boundary if it is a circular arc (NGEOM=1 or 2) or slope of the outer boundary if it is a straight line (NGEOM=3), Fig. III.1.

B0-----R coordinate of the center of curvature of the outer boundary if it is a circular arc (NGEOM=1 or 2) or intercept of the outer boundary if it is a straight line (NGEOM=3), Fig. III.1.

**RC0-----**radius of curvature of the outer boundary if it is a circular arc (NGEOM=1 or 2), Fig. III.1

The next four input variables in NAMELIST PARAM are used to establish a "window" area in the nozzle throat. Subroutine ARMIN searches only this region for the minimum area cross section and subroutine CONTOUR searches only this region for points on the contours of constant Mach number,  $M^*$ , or static-to-stagnation pressure ratio.

**ZIMIN----**minimum Z coordinate on the inner boundary for establishing the throat window, Fig. III.2.

**ZIMAX----**maximum Z coordinate on the inner boundary for establishing the throat window, Fig. III.2.

**Z0MIN----**minimum Z coordinate on the outer boundary for establishing the throat window, Fig. III.2.

**Z0MAX----**maximum Z coordinate on the outer boundary for establishing the throat window, Fig. III.2.

**G-----**specific heat ratio of the gas,  $\gamma$ . (default=1.4)

**ETA-----**parameter  $n$  in the definition of the expansion variable, Eqs. (II-16) or (II-18). (default=2.0)

**NTERM----**number of terms from the expansion solution to be used in evaluating the nozzle discharge coefficient,  $C_D$ . (default=3)

## 2. NAMELIST CONTROL

**NCONT----**an integer variable which indicates the number of contours of constant  $M$ ,  $M^*$ , or  $p/p_0$  to be found. (default=0) NAMELIST NAMECON is read NCONT times.

**START----**a logical variable which if .TRUE. causes a supersonic initial value line to be found for starting method-of-characteristics or finite difference calculations. (default=.FALSE.) If START=.TRUE., NAMELIST NAMEST is read.

NXPL-----an integer variable which specifies the number of planes of constant x coordinate along which flow-field quantities are to be found. (default=0)  
NAMELIST NAMEXPL is read NXPL times.

NZPL-----an integer variable which indicates the number of planes of constant z coordinate along which the flow variables are to be determined. (default=0)  
NAMELIST NAMEZPL is read NZPL times.

### 3. NAMELIST NAMECON

NVAR-----an integer variable which determines which dependent variable is to be held constant along the desired contour. For NVAR=1, the dependent variable is Mach number, M; for NVAR=2, it is the dimensionless speed ratio M\*; and for NVAR=3, it is the static-to-stagnation pressure ratio, p/p<sub>0</sub>. (default=1)

VALUE----the value of the dependent variable along the desired contour.

NPTS-----the number of points to be found along the contour (minimum=4; maximum=53).

NTERM----number of terms from the series expansion solution to be utilized in finding the contour.

### 4. NAMELIST NAMEST

NPTS-----number of points to be found along the supersonic starting line (minimum=4; maximum=53).

NTERM----number of terms from the series solution to be used in determining flowfield quantities along the initial value line.

### 5. NAMELIST NAMEXPL

X-----x coordinate of the plane along which the flowfield quantities are to be determined.

NPTS-----number of points along the plane of constant x coordinate at which the flow variables are to be found (minimum=2; maximum=51).

NTERM----number of terms from the series solution to be utilized in finding the quantities of interest along the x=constant plane.

## 6. NAMELIST NAMEZPL

Z-----z coordinate of the plane along which the flowfield quantities are to be determined.

NPTS----number of points along the plane of constant z coordinate at which the flow variables are to be found (minimum=2; maximum=51).

NTERM----number of terms from the expansion solution to be used in finding the quantities of interest along the z=constant plane.

The first page of output consists of a listing of the parameters from input NAMELISTS PARAM and CONTROL, as just described, together with the following initial quantities determined by subroutines ARMIN and DISCO:

ZI-----z coordinate of the inner boundary throat location,  $z_i$ , Fig. II.1.

RI-----R coordinate of the inner boundary throat location,  $R_i$ , Fig. II.1.

ZO-----z coordinate of the outer boundary throat location,  $z_o$ , Fig. II.1.

RO-----R coordinate of the outer boundary throat location,  $R_o$ , Fig. II.1.

ASTAR----throat area.

D-----separation distance, d, between the inner and outer throat wall locations in the R-Z coordinate system, Fig. II.1.

BETA----angle of inclination,  $\beta$ , of the x-axis from the z-axis of symmetry (positive counterclockwise), Fig. II.1.

YI-----y coordinate of the throat at the inner boundary,  $y_i$ , Fig. II.1.

YO-----y coordinate of the throat at the outer boundary,  $y_o$ , Fig. II.1.

EPS-----value of the expansion variable,  $\epsilon$ , Eq. (II-18).

CD-----nozzle discharge coefficient,  $C_D$ , Eq. (II-33).

The remaining pages of output consist of listings of the parameters in the optional input NAMELISTS NAMECON, NAMEST, NAMEXPL, and NAMEZPL, as used, as well as the following flow-field variables along each contour or plane:

Z-----axial coordinate of the contour point.

R-----radial coordinate of the contour point or of the point on the plane of constant Z coordinate.

Y-----y coordinate of the point on the plane of constant x coordinate.

U-----component of velocity,  $u$ , parallel to the x-axis non-dimensionalized with respect to the critical speed of sound, Eq. (II-5).

V-----component of velocity,  $v$ , parallel to the y-axis non-dimensionalized with respect to the critical speed of sound, Eq. (II-6).

M\*-----dimensionless ratio of the speed at a point to the critical speed of sound.

THETA----angle of inclination of the velocity vector from the x-axis,  $\theta = \tan^{-1}(v/u)$  (positive counterclockwise).

M-----Mach number,  $M$ , which is the dimensionless ratio of the speed at a point to the speed of sound at that point.

P/P<sub>0</sub>-----static-to-stagnation pressure ratio at a point.

### C. INPUT INSTRUCTIONS AND EXAMPLE

The input deck (file) is constructed in the following manner. The first card is a title card which may be any message of up to 80 characters at the user's discretion. This message can be used for identification of both the input file and the

output since it is also the first line of output. The title card is followed by cards containing NAMELISTS PARAM and CONTROL where the usual conventions for reading NAMELISTS are observed. As discussed in the preceding section, the variables in NAMELIST PARAM are geometrical and other parameters necessary in the initialization of the problem, while those in NAMELIST CONTROL are variables which control the further operation of the program. Thus, these NAMELISTS must always appear in the input deck.

The remaining cards in the input file contain, in order, the optional NAMELISTS NAMECON, NAMEST, NAMEXPL, and NAMEZPL. The number of times each of these NAMELIST cards appears in the input deck is determined by the values of the control variables read in NAMELIST CONTROL. NAMELIST NAMECON is read NCONT times; NAMELIST NAMEST is read if START=.TRUE.; NAMELIST NAMEXPL is read NXPL times; and NAMELIST NAMEZPL is read NZPL times. Note that if any of the control variables is left at its default value, the corresponding optional NAMELIST does not appear in the input deck.

Any number of problems can be solved with a single input file by simply repeating the sequence described above. It is important to note, however, that the default values are reset at the beginning of each new problem.

The dimensional variables on input are AI, BI, RCI, A0, B0, RCB, ZIMIN, ZIMAX, Z0MIN, Z0MAX, and z which all have

dimensions of length. The units for these input parameters must be consistent and are simply the units of the R-Z coordinate system. On output these variables, together with  $ZI$ ,  $RI$ ,  $Z\theta$ ,  $R\theta$ , and  $\theta$ , will have these same units and the throat area,  $A_{STAR}$ , will have this unit squared.

As an example consider the annular nozzle shown in Fig. III.3. Both the inner and outer boundaries are circular arcs in the meridional plane so that  $NGEOM=1$ . The centers of curvature of both surfaces lie along the  $Z=0$  plane,  $AI=A\theta=0^\circ$ , with radial coordinates  $SI=-1.625$  and  $S\theta=2.0^\circ$ . The radii of curvature of the inner and outer boundaries are  $RCI=2.0$  and  $RC\theta=1.0$ , respectively.

The input file for this example is shown in Fig. III.4. Following the title card, NAMELIST PARAM is read. In addition to the geometrical parameters just discussed, the throat "window" is set by  $ZIMIN=-0.5$ ,  $ZIMAX=0.5$ ,  $Z\theta MIN=-0.5$ , and  $Z\theta MAX=0.5$ . The remaining variables in PARAM are left at their default values,  $G=1.4$ ,  $ETA=2.0$ , and  $NTERM=3$ . In this example five contours are to be found as well as a supersonic initial value line. Thus, in NAMELIST CONTROL the values  $NCONT=5$  and  $START=.TRUE.$  are specified. Variables  $NXPL$  and  $NZPL$  remain at their default values of zero. The next five cards are used to specify the values of the parameters for each desired contour. In each case 23 points along a constant Mach number contour are desired with three terms from the series solution to be

utilized. Therefore, NVAR=1, NPTS=23, and NTERM=3 are specified. The desired Mach number along the first contour is 0.6 so VALUE=.6. Following the NAMELIST convention, only the variables which are changed need to be specified on successive reads of a given NAMELIST. Therefore, only the different values of the Mach number, VALUE=.8,.8,1.2, and 1.4, are included on the next four NAMECON cards. The final card in the input file specifies that three terms from the expansion solution are to be used to find 22 points along the starting line, NTERM=3 and NPTS=22.

The corresponding output is shown in Fig. III.5. The first page, as discussed in the previous section, consists of the title card, the input variables from NAMELISTS PARAM and CONTROL, and some initialization parameters determined by subroutines ARMIN and DISCO. The next five pages contain listings of the coordinates of the M=0.6, 0.8, 1.0, 1.2, and 1.4 contours as well as the flowfield properties along these contours. The variable NSOLV listed on these pages is the number of contour points actually found. It is included because the contour may pass out of the window area, thus reducing the number of points actually found, and also because if the contour points at  $y_i$  or  $y_o$  are essentially coincident with those on the corresponding boundaries, they are eliminated. The latter is the case for the M=1.0 and 1.2 contours for which NSOLV=22. The last page of output is a listing of the supersonic initial

value line, which in this case is the constant Mach number contour  $M=1.16$  originating from the outer throat wall location.

The Mach number contours listed in Fig. III.5 are plotted in Fig. III.6 together with the corresponding data obtained in [11]. The agreement between the data and the series solution is seen to be quite satisfactory. Note that the values of  $R_{ci}$  and  $R_{co}$  listed in the figure are the radii of curvature of the inner and outer boundaries non-dimensionalized with respect to the throat separation distance,  $d$ , which in this case has a value of 0.625.

#### D. GENERAL DISCUSSION

In addition to annular nozzles, the solution developed in [11] and the TRANNOZ code can be used to analyze throat flow-fields for the simpler cases of planar nozzles and axisymmetric nozzles without centerbodies. However, some care must be exercised in specifying the nozzle geometry for these cases so that the proper results are obtained.

The axisymmetric nozzle configuration is obtained in the limit as the inner boundary of the general annular nozzle approaches the axis of symmetry,  $y_i \rightarrow 0$ , for the case of a straight inner boundary, NGEOM=2. The inner boundary cannot be made to coincide with the axis of symmetry,  $y_i = 0$ , though, since this leads both to division by zero and to zero as the argument of the natural log in the evaluation of the various constants and functions in TRANNOZ. However, the  $y$ -coordinate

of the inner boundary can be made arbitrarily small, e.g.,  $y_i = 10^{-10}$ , thus providing the desired axisymmetric results.

The planar configuration, on the other hand, is obtained in the limit as the dimensionless distance from the axis of symmetry to the inner boundary of the annular nozzle becomes unbounded,  $y_i \rightarrow \infty$ , since the transverse curvature effect becomes negligible in that limit. However, this distance cannot be taken as arbitrarily large in running the TRANNOZ code because of roundoff error considerations. This is due to the fact that the constants and functions in the expansion solution are proportional to powers of  $y$ ,  $y_i$ , and  $y_o$  so that as the latter quantities become large, the evaluation of the desired quantities involves sums and differences of very large, approximately equal quantities. Above certain values of the  $y$  coordinates, roundoff error persists. Table III.1 shows the limiting values of  $y_i$  above which roundoff error affects the solutions for the various orders of both the perturbation velocity components,  $(u_1, v_1)$ ,  $(u_2, v_2)$ , and  $(u_3, v_3)$ , and the discharge coefficient constants,  $C_{D1}$ ,  $C_{D2}$ , and  $C_{D3}$ . These limiting values are shown for both single and double precision versions of subroutine AATRANS, the double precision version being the one routinely used and the one presented here. Notice that the limits on the discharge coefficient constants are stricter than those on the velocity components which is a result of the definition of  $C_D$  as being the integral of the

density-velocity product, Eq. (II-32). The discharge coefficient constants therefore contain higher powers of  $y$ ,  $y_i$ , and  $y_o$  than do the corresponding perturbation velocity components. For the double precision version of AATRANS, Table III.1 shows that for a 60-bit machine and third order solutions, a value no larger than  $y_i = 1000$  should be utilized for investigation of the velocity components and a value no larger than  $y_i = 250$  should be used for determination of the discharge coefficient. Both of these values provide a very good approximation to the planar configuration.

It is to be noted that in analyzing the planar limit any of the three geometrical options, NGEOF=1, 2, or 3, can be used, Fig. III.1. In particular, utilizing NGEOF=1 allows investigation of asymmetric, planar nozzles, i.e., planar nozzles with unequal radii of curvature for the two bounding walls.

Some limitations of the series expansion solution should also be mentioned. During the course of the series solution development, the estimates  $x=0(\epsilon^{1/2})$  and  $\tan\beta/y=0(\epsilon^{3/2})$  were made to satisfy order-of-magnitude consistency requirements in the governing equations. The first estimate implies that the solution is valid only in the transonic region near the throat plane,  $x=0$ . However, as demonstrated in Fig. III.6, the results appear to be quite accurate through a wide region of the throat. The second estimate,  $\tan\beta/y=0(\epsilon^{3/2})$ , means that

for annular nozzles located a small dimensionless distance from the axis of symmetry,  $y=0(1)$ , only small angles of inclination to the axis of symmetry can be analyzed. However, this would seem to be the only physically realistic case anyway, as one would not expect to encounter applications in which the nozzle is both near and highly inclined to the axis of symmetry. As the distance from the axis of symmetry becomes larger, the requirement of small inclination angles can be relaxed. A final limitation is the result of the definition of the expansion parameter,  $\epsilon = (\bar{R}_c + \eta)^{-1}$ . As discussed more fully in [11], the series solution is limited to nozzles whose wall radius of curvature is of the order of the throat separation distance,  $d$ , or larger. For nozzles with  $\bar{R}_c \ll d$ , unrealistic results are obtained.

Despite these limitations, it is felt that the TRANNOZ code provides a fast, inexpensive, and easy-to-use tool for analyzing throat flowfields in a number of nozzle configurations of interest. On a CDC 7600 computer, the compilation time (FTN compiler) for TRANNOZ is approximately 11 seconds while the CPU time required for the sample problem presented in Figs. III.4-III.6 is 1.1 seconds. Also, since the TRANNOZ code was written as a flexible subroutine library, other worker subroutines similar to CONTOUR, STLINE, XPLANE, and ZPLANE can easily be developed to carry out functions not currently included in the program.

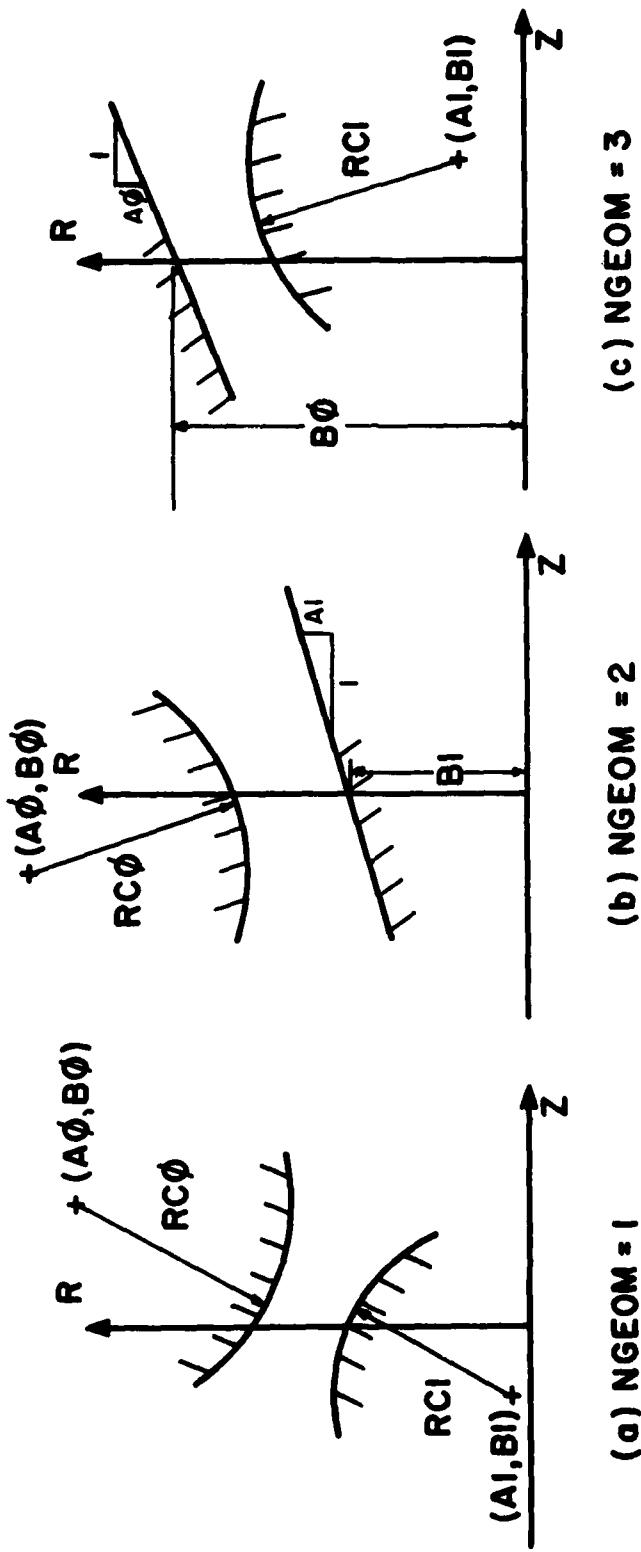


Figure III.1 Sketch depicting input variables NGEOM, AI, BI, RCI,  $A\theta$ ,  $B\theta$ ,  $RC\theta$

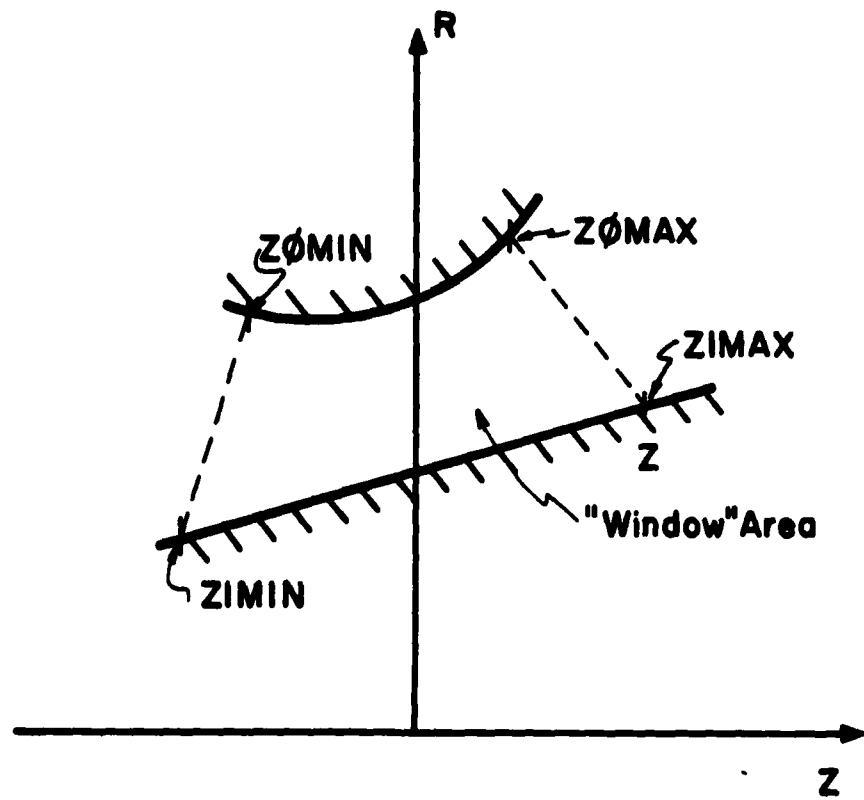


Figure III.2 Sketch depicting input variables ZIMIN, ZIMAX, ZØMIN, ZØMAX

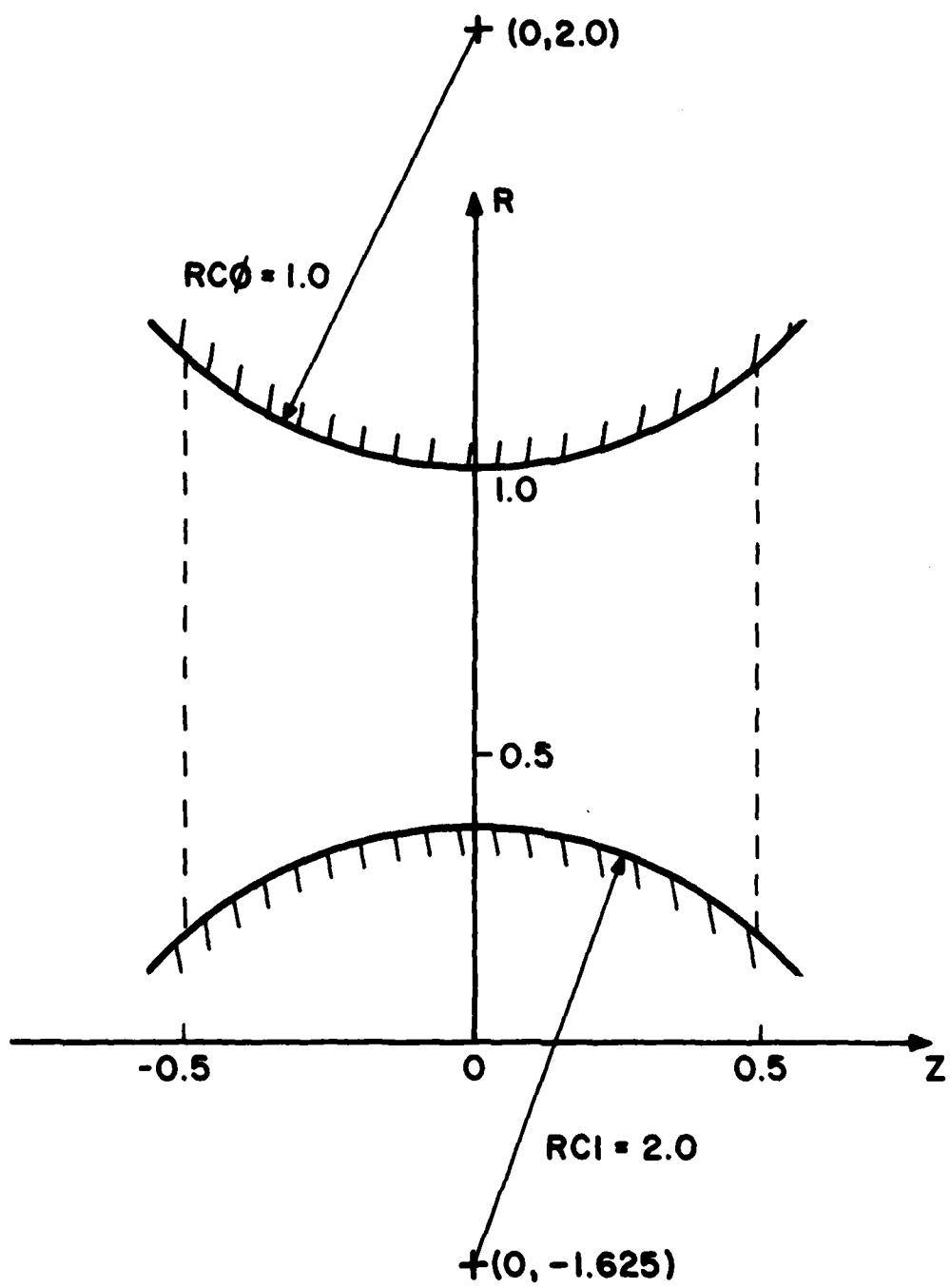


Figure III.3 Geometry of annular nozzle for example input and output

```
EXAMPLE INPUT AND OUTPUT--MACH NUMBER CONTOURS IN AN ANNULAR NOZZLE
$PARAM NGEOM=1,A1=0.0,B1=-1.625,RC1=2.0,A0=0.0,B0=2.0,RC0=1.0,
ZIMIN=-0.5,ZIMAX=0.5,ZOMIN=-0.5,ZOMAX=0.5$
$CONTROL NCNT=5,START=.T.$
$NAMECON NVAR=1,VALUE=0.6,NPTS=23,NTERM=3$
$NAMECON VALUE=0.88
$NAMECON VALUE=1.08
$NAMECON VALUE=1.28
$NAMECON VALUE=1.48
$NAMEST NPTS=22,NTERM=3$
```

Figure III.4 Example input file

RESULTS FROM THE TRANNOZ CODE FOR ANALYZING NOZZLE THROAT FLOWS--BY J. C. DUTTON  
EXAMPLE INPUT AND OUTPUT--MACH NUMBER CONTOURS IN AN ANNULAR NOZZLE

THE GEOMETRY, NGEOM=1, IS A SUPERSONIC NOZZLE WITH:

A CIRCULAR ARC INNER BOUNDARY SUCH THAT IN THE MERIDIONAL PLANE:

AI = ZCENTER = 0.

B1 = RCENTER = -1.6250      RCI = RADIUS = 2.0000

A CIRCULAR ARC OUTER BOUNDARY SUCH THAT IN THE MERIDIONAL PLANE:

AO = ZCENTER = 0.

BO = RCENTER = 2.0000      RCO = RADIUS = 1.0000

THE WINDOW FOR THE THROAT PLANE AND CONTOUR SEARCHES IS SET BY:

ZIMIN = -.50000      ZIMAX = .50000  
ZOMIN = -.50000      ZOMAX = .50000

THE VALUES OF OTHER, NON-GEOMETRICAL PARAMETERS ARE:

G=GAMMA = 1.4000      ETAB = 2.0000

NTERM = 3

THE VALUES OF THE CONTROL VARIABLES ARE:

NCONT = 5      START = T  
NZPL = 0      NZPL = 0

THE INNER AND OUTER WALL THROAT LOCATIONS ARE:

ZI = -.50000E-05      RI = .37500  
ZO = -.25000E-05      RO = 1.0000

THE THROAT AREA, INNER WALL-TO-OUTER WALL THROAT SEPARATION DISTANCE,  
AND ANGLE OF INCLINATION OF THE X-AXIS FROM THE Z-AXIS ARE:

ASTAR = 2.6998      D = .62500      BETA = -.40000E-05

THE Y-CORDINATES OF THE INNER AND OUTER WALL THROAT LOCATIONS AND THE  
VALUE OF THE EXPANSION PARAMETER ARE:

YI = .80000      YO = 1.6000      EPS = .24194

THE VALUE OF THE NOZZLE DISCHARGE COEFFICIENT IS:

CD = .99632

Figure III.5 Example output

DATA FOR A CONTOUR OF CONSTANT MACH NO. (NVAR=1)  
 VALUE= .600000  
 NTERM= 3  
 NSCLV= 23

Z	R	U	V	Hz	THETA	H	P/PO
- .46061	1.1124	.60893	-.37806	.64288	-.48963	.60000	.78985
- .43315	1.00000	.61205	-.26910	.63704	-.37272	.60000	.79224
- .42609	.96875	.61333	-.24557	.63587	-.34454	.60000	.79252
- .41903	.93750	.61484	-.22414	.63495	-.31773	.60000	.79263
- .41186	.90625	.61655	-.20449	.63428	-.29204	.60000	.79256
- .40452	.87500	.61846	-.18632	.63386	-.26730	.60000	.79239
- .39697	.84375	.62052	-.16936	.63367	-.24334	.60000	.79208
- .38925	.81250	.62266	-.15332	.63370	-.22000	.60000	.79167
- .38142	.78125	.62489	-.13796	.63392	-.19717	.60000	.79120
- .37358	.75000	.62710	-.12306	.63430	-.17473	.60000	.79069
- .36587	.71875	.62924	-.10842	.63479	-.15256	.60000	.79016
- .35844	.68750	.63127	-.93832E-01	.63537	-.13056	.60000	.78963
- .35145	.65625	.63315	-.79138E-01	.63600	-.10857	.60000	.78913
- .34509	.62500	.63485	-.64160E-01	.63666	-.86424E-01	.60000	.78866
- .33951	.59375	.63633	-.48715E-01	.63732	-.63926E-01	.60000	.78824
- .33490	.56250	.63756	-.32597E-01	.63797	-.40610E-01	.60000	.78787
- .33140	.53125	.63852	-.15560E-01	.63860	-.16749E-01	.60000	.78756
- .32917	.50000	.63917	.27048E-02	.63918	.86797E-02	.60000	.78729
- .32638	.46675	.63946	.22593E-01	.63971	.36020E-01	.60000	.78709
- .32916	.43750	.63932	.44627E-01	.64020	.65986E-01	.60000	.78695
- .33170	.40625	.63865	.69512E-01	.64064	.99533E-01	.60000	.78666
- .33620	.37500	.63726	.98217E-01	.64102	.13797	.60000	.78636
- .34247	.34546	.63503	.13010	.64133	.18044	.60000	.78690

Figure III.5 Example output (cont.)

DATA FOR A CONTOUR OF CONSTANT MACH NO. (NVAR=1)  
 VALUE= .80000  
 NPTS= 23  
 NTERM= 3  
 NSOLV= 3

Z	R	U	V	M <sub>x</sub>	THETA	M <sub>y</sub>	P/P <sub>0</sub>
-.28306	1.0409	.81549	-.23222	.82664	-.26462	.80000	.65610
-.26264	1.0000	.81704	-.20154	.82622	-.23147	.80000	.65634
-.24785	.96875	.81812	-.18067	.82597	-.20842	.80000	.65648
-.23322	.93750	.81912	-.16175	.82576	-.18725	.80000	.65657
-.21939	.90625	.82006	-.14454	.82555	-.16776	.80000	.65662
-.20620	.87500	.82092	-.12886	.82536	-.14985	.80000	.65665
-.19370	.84375	.82171	-.11450	.82552	-.13329	.80000	.65664
-.18198	.81250	.82243	-.10132	.82552	-.11798	.80000	.65661
-.17110	.78125	.82309	-.89145E-01	.82555	-.10378	.80000	.65656
-.16115	.75000	.82366	-.77948E-01	.82561	-.90545E-01	.80000	.65649
-.15222	.71875	.82421	-.67286E-01	.82569	-.76153E-01	.80000	.65642
-.14440	.68750	.82488	-.57321E-01	.82576	-.66457E-01	.80000	.65634
-.13779	.65625	.82510	-.47808E-01	.82586	-.55301E-01	.80000	.65625
-.13248	.62500	.82546	-.38590E-01	.82598	-.44513E-01	.80000	.65616
-.12856	.59375	.82577	-.29492E-01	.82609	-.33892E-01	.80000	.65606
-.12614	.56250	.82603	-.20306E-01	.82619	-.23204E-01	.80000	.65599
-.12332	.53125	.82624	-.10784E-01	.82630	-.12161E-01	.80000	.65591
-.12020	.50000	.82639	-.61789E-03	.82639	-.41062E-03	.80000	.65584
-.12892	.46875	.82647	.10587E-01	.82646	.12501E-01	.80000	.65577
-.13359	.43750	.82646	.23346E-01	.82657	.27164E-01	.80000	.65570
-.14038	.40625	.82631	.38353E-01	.82665	.44370E-01	.80000	.65564
-.14948	.37500	.82597	.56564E-01	.82672	.65208E-01	.80000	.65559
-.16138	.36926	.82588	.60354E-01	.82673	.69541E-01	.80000	.65556

Figure III.5 Example output (cont.)

DATA FOR A CONTOUR OF CONSTANT MACH NO. (INVAR=1)  
 VALUE = 1.00000  
 NTERM = 3  
 NSOLV = 22  
 NPTS = 23

Z	R	U	V	Hx	THETA	H	P/PO
-111803	1.0088	.99923	-.10259	1.00000	-.10286	1.00000	.52828
-11135	1.0000	.99935	-.98664E-01	1.00000	-.98923E-01	1.00000	.52828
.90243E-01	.99875	.99982	-.82003E-01	1.00000	-.32180E-01	1.00000	.52828
-.76092E-01	.99750	.99918	-.67658E-01	1.00000	-.67773E-01	1.00000	.52828
-.50998E-01	.99625	.99945	-.55429E-01	1.00000	-.55480E-01	1.00000	.52828
-.33090E-01	.97500	.99953	-.45141E-01	1.00000	-.45157E-01	1.00000	.52828
-.16374E-01	.64375	.99977	-.356625E-01	1.00000	-.36613E-01	1.00000	.52828
-.10315E-02	.61250	.99985	-.29723E-01	1.00000	-.29693E-01	1.00000	.52827
.12676E-01	.76125	.99991	-.24283E-01	1.00000	-.24243E-01	1.00000	.52827
.25226E-01	.78000	.99994	-.20157E-01	1.00000	-.20114E-01	1.00000	.52827
.36005E-01	.71875	.99996	-.17202E-01	1.00000	-.17158E-01	1.00000	.52828
.45172E-01	.66750	.99997	-.15276E-01	1.00000	-.15233E-01	1.00000	.52828
.52540E-01	.63625	.99998	-.14234E-01	1.00000	-.14192E-01	1.00000	.52828
.58066E-01	.62500	.99998	-.13930E-01	1.00000	-.13890E-01	1.00000	.52828
.61730E-01	.59375	.99997	-.14206E-01	1.00000	-.14167E-01	1.00000	.52828
.63387E-01	.56250	.99996	-.14867E-01	1.00000	-.14851E-01	1.00000	.52828
.62963E-01	.53125	.99996	-.15768E-01	1.00000	-.15737E-01	1.00000	.52828
.60350E-01	.50000	.99995	-.16602E-01	1.00000	-.16579E-01	1.00000	.52828
.55420E-01	.46875	.99995	-.17068E-01	1.00000	-.17056E-01	1.00000	.52828
.48023E-01	.43750	.99995	-.16741E-01	1.00000	-.16743E-01	1.00000	.52828
.36005E-01	.40625	.99997	-.15035E-01	1.00000	-.15052E-01	1.00000	.52828
.25077E-01	.37500	.99999	-.11092E-01	1.00000	-.11118E-01	1.00000	.52827

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Figure III.5 Example output (cont.)

DATA FOR A CONTOUR OF CONSTANT MACH NO. (INVAR=1)

VALUE= 1.2000

NTERM= 3

NSOLV= 22

NPTS= 23

Z	R	U	V	M*	THETA	M	F/P0
.28878E-01	1.0004	1.1606	.26660E-01	1.1607	.23949E-01	1.2000	.40979
.53524E-01	.96875	1.1602	.37715E-01	1.1604	.33627E-01	1.2000	.41007
.78617E-01	.93750	1.1597	.45741E-01	1.1601	.40545E-01	1.2000	.41034
.10223	.90625	1.1593	.51069E-01	1.1598	.45055E-01	1.2000	.41060
.12634	.87500	1.1589	.53965E-01	1.1596	.47371E-01	1.2000	.41085
.14512	.84375	1.1586	.54572E-01	1.1593	.47701E-01	1.2000	.41107
.16398	.81250	1.1584	.53108E-01	1.1591	.46239E-01	1.2000	.41127
.18097	.78125	1.1583	.49766E-01	1.1590	.43168E-01	1.2000	.41144
.19604	.75000	1.1582	.44729E-01	1.1589	.38656E-01	1.2000	.41157
.20309	.71875	1.1583	.38168E-01	1.1588	.32853E-01	1.2000	.41168
.22004	.68750	1.1583	.30240E-01	1.1587	.25894E-01	1.2000	.41174
.22861	.65625	1.1585	.21087E-01	1.1587	.17896E-01	1.2000	.41177
.23532	.62500	1.1586	.10837E-01	1.1587	.89581E-02	1.2000	.41177
.23949	.59375	1.1587	-.39599E-03	1.1588	-.83018E-03	1.2000	.41173
.24121	.56250	1.1588	-.12500E-01	1.1588	-.11387E-01	1.2000	.41166
.24037	.53125	1.1589	-.25361E-01	1.1589	-.22629E-01	1.2000	.41156
.23683	.50000	1.1589	-.38842E-01	1.1590	-.34454E-01	1.2000	.41143
.23043	.46875	1.1589	-.52759E-01	1.1592	-.46724E-01	1.2000	.41120
.22098	.43750	1.1588	-.66847E-01	1.1594	-.59230E-01	1.2000	.41106
.20828	.40625	1.1587	-.80701E-01	1.1596	-.71639E-01	1.2000	.41082
.19198	.37500	1.1587	-.93692E-01	1.1599	-.83421E-01	1.2000	.41054
.18677	.34375	1.1587	-.97048E-01	1.1600	-.86498E-01	1.2000	.41046

Figure III.5 Example output (cont.)

## DATA FOR A CONTOUR OF CONSTANT MACH NO. (NVAR=1)

VALUE= 1.4000  
NPTS= 23NTERM= 3  
NSOLV= 23

Z	R	U	V	Mx	THETA	M	P/PO
.14570	1.0107	1.3069	.16560	1.3100	.13570	1.4000	.30156
.15661	1.0000	1.3063	.16775	1.3095	.13596	1.4000	.30197
.16758	.96675	1.3046	.17155	1.3064	.13662	1.4000	.30315
.21730	.93750	1.3031	.17161	1.3073	.13746	1.4000	.30452
.24532	.90625	1.3018	.16861	1.3062	.13377	1.4000	.30546
.27160	.87500	1.3008	.16280	1.3052	.12784	1.4000	.30653
.29598	.84375	1.3000	.15406	1.3044	.11991	1.4000	.30754
.31634	.81250	1.2996	.14285	1.3036	.11025	1.4000	.30845
.33857	.78125	1.2993	.12943	1.3029	.99069E-01	1.4000	.30926
.35656	.75000	1.2993	.11404	1.3024	.86563E-01	1.4000	.30995
.37221	.71875	1.2995	.96686E-01	1.3020	.72889E-01	1.4000	.31051
.38541	.68750	1.2998	.78138E-01	1.3017	.56171E-01	1.4000	.31055
.39608	.65625	1.3002	.57931E-01	1.3015	.42468E-01	1.4000	.31125
.40410	.62500	1.3007	.36358E-01	1.3014	.25880E-01	1.4000	.31142
.40937	.59375	1.3011	.13462E-01	1.3014	.83476E-02	1.4000	.31145
.41175	.56250	1.3015	-.10740E-01	1.3015	-.10149E-01	1.4000	.31153
.41108	.53125	1.3017	-.36268E-01	1.3017	-.29681E-01	1.4000	.31106
.40717	.50000	1.3018	-.63153E-01	1.3021	-.50344E-01	1.4000	.31063
.39980	.46875	1.3018	-.91415E-01	1.3026	-.72239E-01	1.4000	.31004
.38869	.43750	1.3016	-.121102	1.3032	-.95443E-01	1.4000	.30926
.37386	.40625	1.3012	-.15161	1.3040	-.11996	1.4000	.30821
.35404	.37500	1.3008	-.18345	1.3050	-.14566	1.4000	.30710
.33264	.34714	1.3006	-.21180	1.3061	-.16922	1.4000	.30567

Figure III.5 Example output (cont.)

## SUPERSONIC STARTING LINE DATA--CONSTANT MACH NUMBER CONTOUR

NPTS= 22  
NTERM= 3  
NSOLV= 22

Z	R	U	V	Mx	Theta	M	P/100
- .25825E-05	1.0000	1.1292	.15905E-05	1.1292	.14268E-05	1.1596	.43282
.25326E-01	.96875	1.1290	.12192E-01	1.1290	.11067E-01	1.1596	.43310
.49463E-01	.93750	1.1287	.21621E-01	1.1288	.19552E-01	1.1596	.43327
.72275E-01	.90625	1.1284	.28491E-01	1.1286	.25671E-01	1.1596	.43344
.93644E-01	.87500	1.1282	.32998E-01	1.1284	.29631E-01	1.1596	.43360
.11345	.84375	1.1280	.35336E-01	1.1283	.31628E-01	1.1596	.43374
.13159	.81250	1.1278	.35692E-01	1.1282	.31850E-01	1.1596	.43387
.14795	.78125	1.1277	.34249E-01	1.1280	.30474E-01	1.1596	.43398
.16246	.75000	1.1276	.31181E-01	1.1280	.27661E-01	1.1596	.43406
.17503	.71875	1.1276	.26652E-01	1.1279	.23564E-01	1.1596	.43412
.18557	.68750	1.1277	.20816E-01	1.1279	.18318E-01	1.1596	.43416
.19402	.65625	1.1277	.13817E-01	1.1279	.12047E-01	1.1596	.43417
.20029	.62500	1.1276	.57855E-02	1.1279	.48611E-02	1.1596	.43417
.20430	.59375	1.1279	.31510E-02	1.1279	-.31342E-02	1.1596	.43414
.20595	.56250	1.1280	-.12867E-01	1.1280	-.11836E-01	1.1596	.43409
.20514	.53125	1.1280	-.23224E-01	1.1280	-.21134E-01	1.1596	.43402
.20174	.50000	1.1280	-.34061E-01	1.1281	-.30833E-01	1.1596	.43395
.19559	.46875	1.1280	-.45160E-01	1.1282	-.40633E-01	1.1596	.43382
.18652	.43750	1.1279	-.56219E-01	1.1284	-.51000E-01	1.1596	.43369
.17431	.40625	1.1279	-.66789E-01	1.1285	-.60700E-01	1.1596	.43355
.15872	.37500	1.1279	-.76196E-01	1.1287	-.69435E-01	1.1596	.43334
.15530	.36896	1.1279	-.77800E-01	1.1288	-.70941E-01	1.1596	.43331

Figure III.5 Example output (cont.)

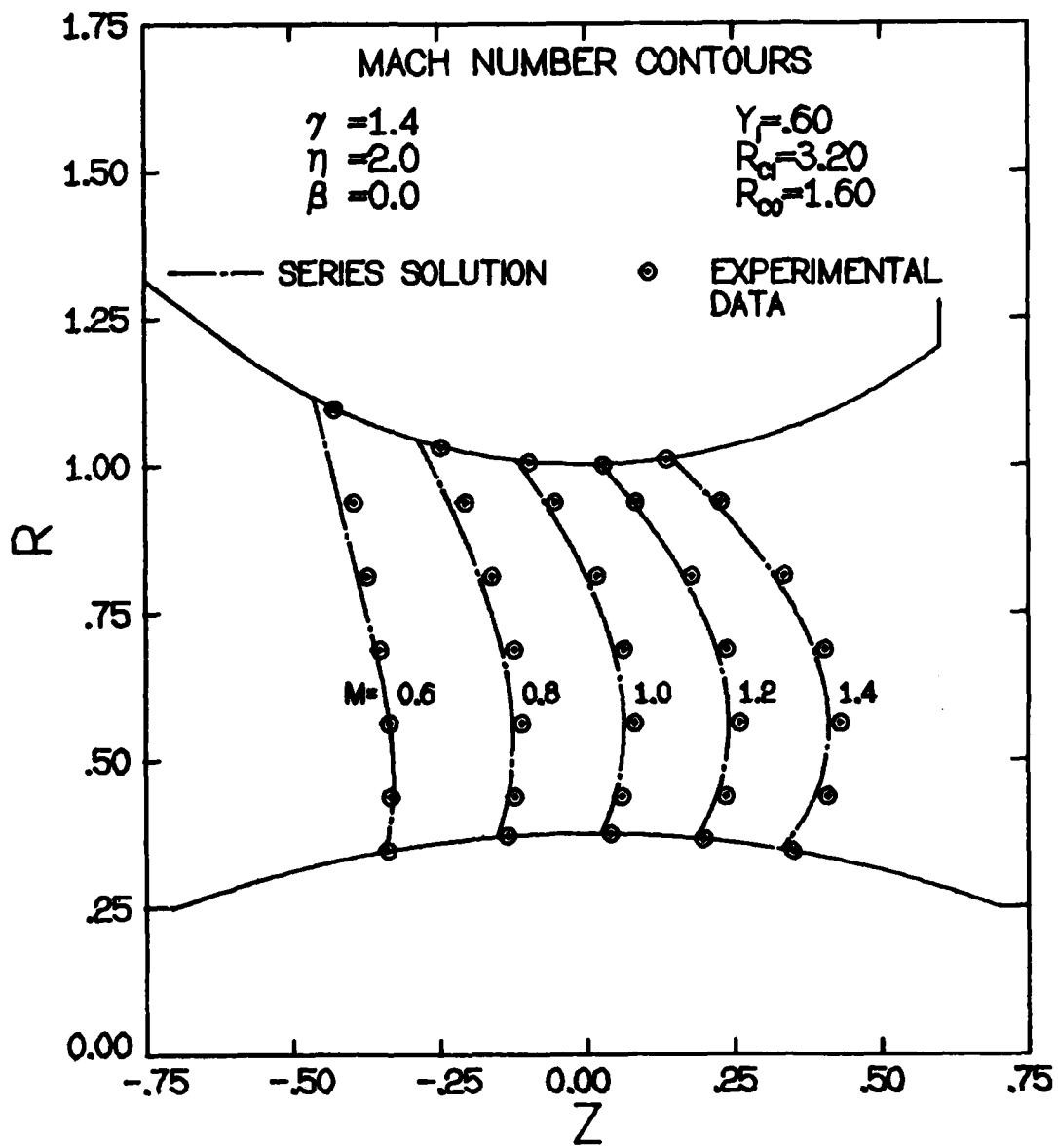


Figure III.6 Comparison of constant Mach number contours from series expansion solution with experimental data for annular nozzle with centerbody center of curvature along  $Z = 0$  plane;  $Re_{2d} = 1.96 \times 10^6$  for experiments

Table III.1 Roundoff Error Investigation

- (a) Approximate values of  $y_i$  at which roundoff error<sup>†</sup> affects solutions for perturbation velocity components:

	Single Precision	Double Precision
First Order $(u_1, v_1)$	3000	$\sim 10^7$
Second Order $(u_2, v_2)$	60	30,000
Third Order $(u_3, v_3)$	15	1000

- (b) Approximate values of  $y_i$  at which roundoff error<sup>†</sup> affects solutions for discharge coefficient constants:

	Single Precision	Double Precision
First Order $(C_{D1})$	120	50,000
Second Order $(C_{D2})$	20	1500
Third Order $(C_{D3})$	10	250

<sup>†</sup>On the University of Illinois CDC Cyber 175 digital computer

#### IV. CONCLUSIONS

A FORTRAN computer program, TRANNOZ, has been developed to analyze the transonic throat flowfields in annular, planar, and axisymmetric supersonic nozzles. The program evaluates the series expansion solution developed by Dutton and Addy in [11]. Among its capabilities are options to find contours of constant Mach number,  $M^*$ , or static-to-stagnation pressure, to calculate an accurate initial value line for starting method-of-characteristics or finite difference calculations, and to determine flowfield quantities along various planes in the nozzle throat. Major features of TRANNOZ are its numerical speed and reliability so that computations can be carried out easily and routinely.

The functioning of the various subroutines in the code has been described together with the definitions of the input and output variables, detailed input instructions, and an example input file with the corresponding output. A brief description of the theory upon which the solution is based has also been included.

As a result of the characteristics described above, it is felt that the TRANNOZ program provides an efficient means for obtaining approximate solutions for the flowfields in the throat regions of a variety of supersonic nozzles of interest.

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## TRANNOZ CODE...PROGRAM MAIN

PROGRAM MAIN INPUT, OUTPUT, TAPES, INPUT/TAPES/OUTPUT)

MR.HETHERIVY: VTPR/MAE/4/10/78

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 FEBRUARY 5 1980 3N 251.14.GN 1C 55  
 BY E231

PROGRAM MAIN IS THE MAIN PROGRAM IN THE TRANNOZ CODE WHICH SIMULATES TRANSonic FLOW IN THE THROAT REGION OF ANULAR, AXI-SYMMETRIC, AND PLANAR SUPERSONIC NOZZLES. THE SEVEN SUBROUTINES ARE DEVELOPED BY DUTTON (PH.D. THESIS, UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN, 1978) AND USED TO EVALUATE THE FLOWFIELD VARIABLES OF INTEREST. THE MAIN PROGRAM SIMPLY READS THE INPUT PARAMETERS, CALLS SUBROUTINES ARRANGED AS FOLLOWS, CALCULATES THE DESIRED FLOWFIELD QUANTITIES, AND WRITES THE RESULTS.

THE INPUT VARIABLES FROM FILE INPUT ARE AS FOLLOWS.

FIRST FROM NAMELIST PARAMETER:

- \*NREON--AN INTEGER VARIABLE DEFINING THE NUMBER OF REONS IN THE FLOWFIELD.
- \*IF NREON=1, BOTH THE INNER AND OUTER BOUNDARIES ARE CIRCULAR, AND THE THROAT LINE IS A STRAIGHT LINE. IF NREON=2, THE INNER BOUNDARY IS A CIRCULAR ARC, WHILE THE OUTER BOUNDARY IS A CIRCULAR ARC. IF NREON=3, THE INNER BOUNDARIES OF CIRCULAR ARCS AND THE OUTER BOUNDARY IS A STRAIGHT LINE (DEFAULP T 1). IN EACH CASE,
- \*1--COORDINATE OF THE CENTER OF CURVATURE OF THE INNER BOUNDARY FOR A CIRCULAR ARC (NREON=1 OR 2) OR COORDINATE OF THE CENTER OF CURVATURE OF THE INNER BOUNDARY FOR A CIRCULAR ARC (NREON=1 OR 3) OR INTERCEPT OF THE INNER BOUNDARY FOR A STRAIGHT LINE (NREON=2).
- \*2--RADIUS OF THE INNER BOUNDARY (NREON=1 OR 2) OR LENGTH OF THE INNER BOUNDARY (NREON=3).
- \*3--EXponent OF THE INNER BOUNDARY ARC LENGTH (NREON=1 OR 2).
- \*4--COORDINATE OF THE CENTER OF CURVATURE OF THE OUTER BOUNDARY FOR A CIRCULAR ARC (NREON=1 OR 2) OR SLOPE OF THE OUTER BOUNDARY FOR A STRAIGHT LINE (NREON=3).
- \*5--COORDINATE OF THE CENTER OF CURVATURE OF THE OUTER BOUNDARY FOR A CIRCULAR ARC (NREON=1 OR 2) OR INTERCEPT OF THE OUTER BOUNDARY FOR A STRAIGHT LINE (NREON=3).
- \*6--RADIUS OF THE OUTER BOUNDARY FOR A CIRCULAR ARC (NREON=1 OR 2).
- \*7--MINIMUM Z COORDINATE ON THE INNER BOUNDARY FOR ESTABLISHING THE THROAT "WINDON".

## APPENDIX. TRANNOZ PROGRAM LISTING

TRANNOZ CODE...PROGRAM MAIN (CONT.)

```

C...      ZIMAX---MAXIMUM Z COORDINATE ON THE INNER BOUNDARY FOR
C...      ESTABLISHING THE THROAT "WINDOW"
C...      ZMIN---MINIMUM Z COORDINATE ON THE OUTER BOUNDARY FOR
C...      ESTABLISHING THE THROAT "WINDOW"
C...      ZMAX---MAXIMUM Z COORDINATE ON THE OUTER BOUNDARY FOR
C...      ESTABLISHING THE THROAT "WINDOW"
C...      0----SPECIFIC HEAT RATIO OF THE GAS. (DEFAULT=1.4)
C...      ETA----PARAMETER IN THE EXPANSION VARIABLE (DEFAULT=2.0)
C...      NTERM---NUMBER OF TERMS FROM THE EXPANSION SOLUTION TO BE
C...      USED IN EVALUATING THE NOZZLE DISCHARGE COEFFICIENT
C...      (DEFAULT=3)

C...FROM NAMELIST CONTROL:
C...      NCNT---NUMBER OF CONTOURS OF CONSTANT MACH NUMBER, MSTAR,
C...      OR P/PO TO BE FOUND (DEFAULT=0)
C...      START---LOGICAL VARIABLE WHICH IF .TRUE. CAUSES A SUPERSONIC
C...      STARTING LINE FOR METHOD-OF-CHARACTERISTICS (OR
C...      OTHER) CALCULATIONS TO BE FOUND (DEFAULT=.FALSE.)
C...      NXPL---NUMBER OF PLANES OF CONSTANT X COORDINATE ALONG
C...      WHICH FLOWFIELD QUANTITIES ARE TO BE FOUND
C...      (DEFAULT=0)
C...      NZPL---NUMBER OF PLANES OF CONSTANT Z COORDINATE ALONG
C...      WHICH FLOWFIELD QUANTITIES ARE TO BE FOUND
C...      (DEFAULT=0)

C...FROM NAMELIST NAMECON:
C...      NVAR---AN INTEGER VARIABLE WHICH DETERMINES WHICH DEPENDENT
C...      VARIABLE IS CONSTANT ALONG THE DESIRED CONTOUR. FOR
C...      NVAR=1, THE DEPENDENT VARIABLE IS MACH NUMBER, WHILE
C...      FOR NVAR=2 IT IS MSTAR, AND FOR NVAR=3 IT IS P/PO.
C...      (DEFAULT=1)
C...      VALUE---THE VALUE OF THE DEPENDENT VARIABLE ALONG THE DESIRED
C...      CONTOUR
C...      NPTS---THE NUMBER OF POINTS TO BE FOUND ALONG THE DESIRED
C...      CONTOUR (MIN.=4; MAX.=153)
C...      NTERM---NUMBER OF TERMS FROM THE EXPANSION SOLUTION TO BE
C...      USED IN FINDING THE CONTOUR

C...FROM NAMELIST NAMEST:
C...      NPTS---NUMBER OF POINTS TO BE FOUND ALONG THE SUPERSONIC
C...      STARTING LINE (MIN.=4; MAX.=153)
C...      NTERM---NUMBER OF TERMS FROM THE EXPANSION SOLUTION TO BE
C...      USED IN DETERMINING FLOWFIELD QUANTITIES ALONG THE
C...

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TRANNOZ CODE...PROGRAM MAIN (CONT.)

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C... FROM NAMELIST NAMEXPL:                               MAI 970
C... X-----X COORDINATE OF THE PLANE ALONG WHICH FLOWFIELD      MAI 980
C... QUANTITIES ARE TO BE FOUND                                MAI 990
C... NPTS----NUMBER OF POINTS ALONG THE CONSTANT X PLANE AT WHICH    MAI 1010
C... FLOWFIELD QUANTITIES ARE TO BE FOUND (MIN.=2;                  MAI 1020
C... MAX.=51)                                                 MAI 1030
C... NTERM---NUMBER OF TERMS FROM THE EXPANSION SOLUTION TO BE      MAI 1040
C... USED IN FINDING QUANTITIES ALONG THE X=CONSTANT                MAI 1050
C... PLANE                                                 MAI 1060
C... MAI 1070

C... FROM NAMELIST NAMEZPL:                               MAI 1080
C... Z-----Z COORDINATE OF THE PLANE ALONG WHICH FLOWFIELD      MAI 1090
C... QUANTITIES ARE TO BE FOUND                                MAI 1100
C... NPTS----NUMBER OF POINTS ALONG THE CONSTANT Z PLANE AT WHICH    MAI 1120
C... FLOWFIELD QUANTITIES ARE TO BE FOUND (MIN.=2;                  MAI 1130
C... MAX.=51)                                                 MAI 1140
C... NTERM---NUMBER OF TERMS FROM THE EXPANSION SOLUTION TO BE      MAI 1150
C... USED IN FINDING QUANTITIES ALONG THE Z=CONSTANT                MAI 1160
C... PLANE                                                 MAI 1170
C... MAI 1180

C... THE INPUT FILE IS CONSTRUCTED AS FOLLOWS... THE FIRST CARD IS A      MAI 1190
C... TITLE CARD (MAXIMUM OF 80 CHARACTERS) TO BE USED FOR IDENT-      MAI 1200
C...IFICATION PURPOSES. ANY MESSAGE MAY BE USED AND THIS MESSAGE      MAI 1210
C... IS THE FIRST LINE ON THE OUTPUT. THIS CARD IS FOLLOWED BY      MAI 1220
C... CARDS CONTAINING NAMELIST PARAM AND NAMELIST CONTROL, RESPEC-      MAI 1230
C... TIVELY. THESE TWO NAMELISTS MUST ALWAYS APPEAR IN THE INPUT      MAI 1240
C... DECK. THE REMAINING CARDS IN THE INPUT FILE CONTAIN, IN ORDER,      MAI 1250
C... NAMELISTS NAMECON, NAMEPL, AND NAMEZPL, ALTHOUGH      MAI 1260
C... SOME OF THESE NAMELISTS MAY BE REPEATED, AND SOME MAY NOT      MAI 1270
C... APPEAR AT ALL. NAMELIST NAMECON IS REPEATED NCNT TIMES;      MAI 1280
C... NAMELIST NAMEST APPEARS ONLY IF START=.TRUE.; NAMELIST NAMEXPL      MAI 1290
C... IS REPEATED NZPL TIMES; AND NAMELIST NAMEZPL IS REPEATED NZPL      MAI 1300
C... TIMES SINCE NCNT, NXPL, AND NZPL MAY BE ZERO. NAMELISTS      MAI 1310
C... NAMECON, NAMEPL, AND NAMEZPL DO NOT NECESSARILY APPEAR IN THE      MAI 1320
C... INPUT FILE. ANY NUMBER OF PROBLEMS CAN BE SOLVED WITH A SINGLE      MAI 1330
C... INPUT FILE BY REPEATING THE SEQUENCE DESCRIBED ABOVE. IT IS      MAI 1340
C... IMPORTANT TO NOTE, HOWEVER, THAT THE DEFAULT VALUES ARE RESET      MAI 1350
C... AT THE BEGINNING OF EACH NEW PROBLEM.                         MAI 1360
C... MAI 1370

C... THE FIRST PAGE OF OUTPUT CONSISTS OF THE TITLE CARD, A LISTING      MAI 1380
C... OF THE PARAMETERS FROM INPUT NAMELISTS PARAM AND CONTROL, AS      MAI 1390
C... WELL AS THE FOLLOWING VARIABLES DESCRIBING THE THROAT GEOMETRY:      MAI 1400
C... MAI 1410
C... Z1-----Z COORDINATE OF THE THROAT AT THE INNER BOUNDARY      MAI 1420
C... RI-----R COORDINATE OF THE THROAT AT THE INNER BOUNDARY      MAI 1430
C... Z0-----Z COORDINATE OF THE THROAT AT THE OUTER BOUNDARY      MAI 1440
C... RO-----R COORDINATE OF THE THROAT AT THE OUTER BOUNDARY      MAI 1450
C... MAI 1460

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TRANNOZ CODE...PROGRAM MAIN (CONT.)

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C... ASTAR---THROAT AREA
C... D-----SEPARATION DISTANCE BETWEEN THE INNER AND OUTER THROAT
C... C... WALL LOCATIONS IN THE Z-R COORDINATE SYSTEM
C... C... BETA---ANGLE OF INCLINATION OF THE ROTATED X-AXIS FROM THE
C... C... Z-AXIS OF SYMMETRY (POSITIVE COUNTERCLOCKWISE)
C... C... YI-----Y COORDINATE OF THE THROAT AT THE INNER BOUNDARY
C... C... YO-----Y COORDINATE OF THE THROAT AT THE OUTER BOUNDARY
C... C... EPS---VALUE OF THE EXPANSION VARIABLE
C... C... CD-----NOZZLE DISCHARGE COEFFICIENT
C... MA1470
C... MA1480
C... MA1490
C... MA1500
C... MA1510
C... MA1520
C... MA1530
C... MA1540
C... MA1550
C... MA1560
C... MA1570
C... MA1580
C... MA1590
C... MA1600
C... MA1610
C... MA1620
C... MA1630
C... MA1640
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C... MA1660
C... MA1670
C... MA1680
C... MA1690
C... MA1700
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C... MA1780
C... MA1790
C... MA1800
C... MA1810
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C... MA1830
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C... MA1850
C... MA1860
C... MA1870
C... MA1880
C... MA1890
C... MA1900
C... MA1910
C... MA1920
C... MA1930
C... MA1940
C... MA1950

C... THE REMAINING PAGES OF OUTPUT CONSIST OF LISTINGS OF THE PARA-
C... METERS FROM THE OPTIONAL INPUT NAMELISTS NAMECON, NAMEST,
C... NAMEXPL, NAMEZPL, AS USED, TOGETHER WITH THE FOLLOWING VARI-
C... ABLES ALONG THE CONTOUR OR PLANE:
C... MA1160
C... MA11610
C... MA11620
C... MA11630
C... MA11640
C... MA11650
C... MA11660
C... MA11670
C... MA11680
C... MA11690
C... MA11700
C... MA11710
C... MA11720
C... MA11730
C... MA11740
C... MA11750
C... MA11760
C... MA11770
C... MA11780
C... MA11790
C... MA11800
C... MA11810
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C... MA11860
C... MA11870
C... MA11880
C... MA11890
C... MA11900
C... MA11910
C... MA11920
C... MA11930
C... MA11940
C... MA11950

C... 2-----AXIAL COORDINATE OF THE CONTOUR POINT
C... R-----RADIAL COORDINATE OF THE CONTOUR POINT OR OF THE
C... POINT ON THE Z=CONSTANT PLANE
C... C... Y-----ROTATED RADIAL COORDINATE OF THE POINT ON THE X=
C... CONSTANT PLANE (THE Y-AXIS LIES ALONG THE THROAT
C... PLANE AND THE X-AXIS IS PERPENDICULAR TO IT SUCH
C... THAT THE ORIGIN LIES ON THE Z-AXIS OF SYMMETRY.
C... THE X AND Y COORDINATES ARE NON-DIMENSIONALIZED WITH
C... RESPECT TO THE THROAT SEPARATION DISTANCE, D.)
C... U-----COMPONENT OF VELOCITY PARALLEL TO THE X-AXIS NON-
C... DIMENSIONALIZED WITH RESPECT TO THE CRITICAL SPEED
C... OF SOUND
C... V-----COMPONENT OF VELOCITY PARALLEL TO THE Y-AXIS NON-
C... DIMENSIONALIZED WITH RESPECT TO THE CRITICAL SPEED
C... OF SOUND
C... M-----DIMENSIONLESS RATIO OF THE SPEED AT A POINT TO THE
C... CRITICAL SPEED OF SOUND
C... C... THETA---ANGLE OF INCLINATION OF THE VELOCITY VECTOR FROM
C... THE X-AXIS, THETA=ARCTAN(V/U) (POSITIVE COUNTER-
C... CLOCKWISE)
C... M-----MACH NUMBER=DIMENS! ONLESS RATIO OF THE SPEED AT A
C... POINT TO THE SPEED OF SOUND AT THAT POINT
C... P/PO---STATIC-TO-STAGNATION PRESSURE RATIO AT A POINT
C... C... ON INPUT THE DIMENSIONAL VARIABLES ARE AI, BI, RCI, AO, BO,
C... RCO, ZIMIN, ZIMAX, ZOMIN, AND Z WHICH ALL HAVE DIMENSIONS
C... OF LENGTH. THE UNITS USED FOR THESE INPUT PARAMETERS MUST
C... BE CONSISTENT AND ARE SIMPLY THE UNITS OF THE Z-R COORDINATE
C... SYSTEM. ON OUTPUT THESE VARIABLES TOGETHER WITH ZI, RI, ZO,
C... RO, AND D WILL HAVE THE SAME UNITS AND ASTAR WILL HAVE THIS
C... UNIT SQUARED.
C... REAL MSCONT, MCINT, MSXPL, MXPL, MSZPL, MZPL
C... LOGICAL START

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TRANNOZ CODE... PROGRAM MAIN (CONT.)

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COPION/BLKIN/S, ETA, ZIMIN, ZIMAX, ZOMIN, ZOMAX/BLKCIRG/NGEOM,
SAI, BI, RCI, AO, BO, RCO/BLKBDM/R1, Z1, RD, ASTAR, D, ZSTAR, BETA,
SHIP, G1P, HEP, G2P/BLKPARN/Y1, YO, EPS, H1, G1, H2, G2, BETAI
S/BLKCONT/NSOLV, RCONT(53), ZCONT(53), UCONT(53), VCONT(53),
SHSCONT(53), THCONT(53), MCONT(53), PCONT(53)/BLKXPL/THPL(51),
SUXPL(51), VXPPL(51), MSXPL(51), TXPL(51), NXPL(51), PXPL(51),
SU1XPL(51), V1XPL(51), U2XPL(51), V2XPL(51), USXPL(51), VSXPL(51),
S/BLKZPL/RZPL(51), UZPL(51), VZPL(51), NSZPL(51), TSZPL(51),
SZPL(51), PP0ZPL(51), U1ZPL(51), V1ZPL(51), U2ZPL(51), V2ZPL(51),
SU3ZPL(51), V3ZPL(51)/BLKCALL/ICALL1, ICALL2, ICALL3
DIMENSION TITLE(8)
NAMELIST/ST/PARAM/NGEOM, AI, BI, RCI, AO, BO, RCO, ZIMIN, ZIMAX,
ZOMIN, ZOMAX, NPTS, NTERM, NAMEPL/CONTROL, START, NXPL, NZPL/NAMECON/
SNVAR, VALUE, NPTS, NTERM, NAMEST/NPTS, NTERM, NAMEPL/X, NPTS,
SNTERM/NAMEZPL/Z, NPTS, NTERM

C... SET DEFAULT VALUES:
C
      10  NOECM=1      S  0#1.4   S  ETAB2.0
      11  NTERM=3      S  NCONT=0   S  START=.FALSE.
      12  NXPL=0       S  NZPL=0   S  NVAR=1
      13  ICALL1=0     S  ICALL2=0  S  ICALL3=0

C... READ AND WRITE THE TITLE AND READ AND WRITE THE INPUT PARA-
C... METERS FROM NAMELISTS PARAM AND CONTROL:
C
      14  READ(5,900)  TITLE(1), I=1,8
      15  IF(IEOF(5))  170,20
      16  WRITE(6,901)  (TITLE(I), I=1,8)
      17  READ(5,PARAM)
      18  WRITE(6,902)  NGEOM
      19  GO TO 30,40,50,60,70,80,90,100
      20  WRITE(6,903)  AI,BI,RCI
      21  WRITE(6,904)  AO,BO,RCO
      22  GO TO 60
      23  WRITE(6,905)  AI,BI
      24  WRITE(6,904)  AO,BO,RCO
      25  GO TO 60
      26  WRITE(6,903)  AI,BI,RCI
      27  WRITE(6,906)  AO,BO
      28  WRITE(6,907)  ZIMIN,ZIMAX,ZOMIN,ZOMAX
      29  WRITE(6,908)  G,ETA,NTERM
      30  READ(5,CONTROL)
      31  WRITE(6,909)  NCONT,START,NXPL,NZPL

C... CALL SUBROUTINE ARMIN TO LOCATE THE THROAT PLANE AND CALCULATE
C... AND WRITE SOME INITIAL PARAMETERS:
C
      32  CALL ARMIN
      33  WRITE(6,910)  Z1,RI,Z0,RO
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TRANNOZ CODE... PROGRAM MAIN (CONT.)

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      WRITE(6,911) ASTAR,D,BETA
      WRITE(6,912) Y1,Y0,EPS
C... CALL SUBROUTINE DISCO TO CALCULATE THE DISCHARGE COEFFICIENT
C... AND WRITE IT:
C   CALL DISCO(NTERM,FLWCO)
      WRITE(6,913) FLWCO

C... READ, CALCULATE, AND WRITE CONTOUR DATA:
C
      IF(NCONT .EQ. 0) GO TO 120
      DO 110 J=1,NCONT
      READ(5,NAMECON)
      GO TO(70,80,90),NVAR
      70  VARIAB="MACH NO." S  GO TO 100
      80  VARIAB="MSTAR"  S  GO TO 100
      90  VARIAB="P/PO"
100   CALL CONTOUR(NVAR,VALUE,NPTS,NTERM)
      WRITE(6,914) VARIAB,NVAR,VALUE,NTERM,NPTS,NSOLV
      LIM=23
      IF(INSOLV .LE. 23) LIM=NSOLV
      WRITE(6,915) (ZCONT(I),RCONT(I),UCONT(I),VCONT(I),MSCONT(I),
      STHCONT(I),MCONT(I),PPOCONT(I),I=1,LIM)
      IF(LIM .EQ. NSOLV) GO TO 110
      LIM=49
      IF((NSOLV-23) .LE. 26) LIM=NSOLV
      WRITE(6,916)
      WRITE(6,915) (ZCONT(I),RCONT(I),UCONT(I),VCONT(I),MSCONT(I),
      STHCONT(I),MCONT(I),PPOCONT(I),I=1,LIM)
      IF(LIM .EQ. NSOLV) GO TO 110
      WRITE(6,916)
      WRITE(6,915) (ZCONT(I),RCONT(I),UCONT(I),VCONT(I),MSCONT(I),
      STHCONT(I),MCONT(I),PPOCONT(I),I=50,NSOLV)
110   CONTINUE
C
C... READ, CALCULATE, AND WRITE STARTING LINE DATA:
C
120   IF(.NOT.(START)) GO TO 130
      READ(5,NAMEST)
      CALL STLINE(NPTS,NTERM)
      WRITE(6,917) NPTS,NSOLV,NTERM
      LIM=23
      IF(INSOLV .LE. 23) LIM=NSOLV
      WRITE(6,915) (ZCONT(I),RCONT(I),UCONT(I),VCONT(I),MSCONT(I),
      STHCONT(I),MCONT(I),PPOCONT(I),I=1,LIM)
      IF(LIM .EQ. NSOLV) GO TO 130
      LIM=49
      IF((NSOLV-23) .LE. 26) LIM=NSOLV
      WRITE(6,916)

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TRANNOZ CODE... PROGRAM MAIN (CONT.)

```

      WRITE(6,915) (ZCINT(I),RCINT(I),UCINT(I),VCINT(I),MSINT(I),
      $THCINT(I),MCINT(I),PPOCINT(I),I=24,LIM)
      IF(LIM .EQ. NSOLV) GO TO 130
      WRITE(6,916)
      WRITE(6,915) (ZCINT(I),RCINT(I),UCINT(I),VCINT(I),MSINT(I),
      $THCINT(I),MCINT(I),PPOCINT(I),I=50,NSOLV)

C... READ, CALCULATE, AND WRITE DATA ALONG PLANES OF CONSTANT X
C... COORDINATE:
C 130 IF(NXPL .EQ. 0) GO TO 150
      DO 140 L=1,NXPL
      READ(5,NAMEXPL)
      CALL XPLANE(X,NPTS,INTERM)
      WRITE(6,918) X,NPTS,INTERM
      LIM=23
      IF(NPTS .LE. 23) LIM=NPTS
      WRITE(6,919) (YXPL(I),UXPL(I),VXPL(I),MSXPL(I),THXPL(I),
      $MXPL(I),PPOXPL(I),I=1,LIM)
      IF(LIM .EQ. NPTS) GO TO 140
      LIM=49
      IF((NPTS-23) .LE. 26) LIM=NPTS
      WRITE(6,916)
      WRITE(6,919) (YXPL(I),UXPL(I),VXPL(I),MSXPL(I),THXPL(I),
      $MXPL(I),PPOXPL(I),I=24,LIM)
      IF(LIM .EQ. NPTS) GO TO 140
      WRITE(6,916)
      WRITE(6,919) (YXPL(I),UXPL(I),VXPL(I),MSXPL(I),THXPL(I),
      $MXPL(I),PPOXPL(I),I=50,NPTS)
      140 CONTINUE

C... READ, CALCULATE, AND WRITE DATA ALONG PLANES OF CONSTANT Z
C... COORDINATE:
C 150 IF(NZPL .EQ. 0) GO TO 160
      DO 160 L=1,NZPL
      READ(5,NAMEZPL)
      CALL ZPLANE(Z,NPTS,INTERM)
      WRITE(6,920) Z,NPTS,INTERM
      LIM=23
      IF(NPTS .LE. 23) LIM=NPTS
      WRITE(6,921) (RZPL(I),UZPL(I),VZPL(I),MSZPL(I),THZPL(I),
      $MZPL(I),PPOZPL(I),I=1,LIM)
      IF(LIM .EQ. NPTS) GO TO 160
      LIM=49
      IF((NPTS-23) .LE. 26) LIM=NPTS
      WRITE(6,921) (RZPL(I),UZPL(I),VZPL(I),MSZPL(I),THZPL(I),
      $MZPL(I),PPOZPL(I),I=24,LIM)
      IF(LIM .EQ. NPTS) GO TO 160

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TRANNOZ CODE... PROGRAM MAIN (CONT.)

```

      WRITE(6,916)
      WRITE(6,921) (RZPL(1),UZPL(1),VZPL(1),MSZPL(1),THZPL(1))
      SMZPL(1),PP0ZPL(1),I=50,NPTS)
160  CONTINUE
      GO TO 10

C...FORMAT STATEMENTS:
C
 900  FORMAT(8A10)
 901  FORMAT(1H1, //, 20X, "RESULTS FROM THE TRANNOZ CODE FOR ANALY",
     S"2ING NOZZLE THROAT FLOWS --BY J. C. DUTTON", //, 25X, 8A10, //)
 902  FORMAT(20X, "THE GEOMETRY, NGEOM=", 11, -, IS A SUPERSONIC",
     S"NOZZLE WITH:", /)
 903  FORMAT(25X, "A CIRCULAR ARC INNER BOUNDARY SUCH THAT IN",
     S"THE MERIDIONAL PLANE:", //, 25X, "A1=ZCENTER=", 811.5, SX,
     S"BI=RCENTER=", 811.5, SX, "RC1=RADIUS=", G11.5, /)
 904  FORMAT(25X, "A CIRCULAR ARC OUTER BOUNDARY SUCH THAT IN",
     S"THE MERIDIONAL PLANE:", //, 25X, "AO=ZCENTER=", 811.5, SX,
     S"BO=RCENTER=", 811.5, SX, "RC0=RADIUS=", G11.5, /)
 905  FORMAT(25X, "A STRAIGHT INNER BOUNDARY SUCH THAT IN THE",
     S"MERIDIONAL PLANE:", //, 25X, "AI=SLOPE=", 811.5, 7X, "BI=", ,
     S"INTERCEPT=", G11.5, /)
 906  FORMAT(25X, "A STRAIGHT OUTER BOUNDARY SUCH THAT IN THE",
     S"MERIDIONAL PLANE:", //, 25X, "AO=SLOPE=", 811.5, 7X, "BO=", ,
     S"INTERCEPT=", G11.5, /)
 907  FORMAT(20X, "THE WINDOW FOR THE THROAT PLANE AND CONTOUR",
     S"SEARCHES IS SET BY:", //, 25X, "ZIMIN=", 811.5, 10X, "ZIMAX=",
     S"Z11.5, /, 25X, "ZOMIN=", G11.5, 10X, "ZOMAX=", G11.5, //)
 908  FORMAT(20X, "THE VALUES OF OTHER, NON-GEOMETRICAL PARAM",
     S"ETERS ARE: ", //, 25X, "Q=BAMMA=", 811.5, 8X, "ETA=", , G11.5, 12X,
     S"INTERM=", 12, //)
 909  FORMAT(20X, "THE VALUES OF THE CONTROL VARIABLES ARE: ", //,
     S"Z25X, "NCONT=", 12, 19X, "START=", L2, /, 25X, "NXPL=", , 12, 20X,
     S"NZPL=", 12, /)
 910  FORMAT(20X, "THE INNER AND OUTER WALL THROAT LOCATIONS ARE:",
     S"/, 25X, "Z1=", 811.5, 13X, "R1=", 811.5, /, 25X, "Z0=", 811.5, 13X,
     S"R0=", G11.5, /)
 911  FORMAT(20X, "THE THROAT AREA, INNER WALL-TO-OUTER WALL",
     S"SEPARATION DISTANCE, ", /, 20X, "AND ANGLE OF INCLINA",
     S"TION OF THE X-AXIS FROM THE Z-AXIS ARE: ", //, 25X, "ASTAR=",
     S"811.5, 10X, "D=", G11.5, 14X, "BETA=", G11.5, //)
 912  FORMAT(20X, "THE Y-CORDINATES OF THE INNER AND OUTER WALL",
     S"THROAT LOCATIONS AND THE", /, 20X, "VALUE OF THE EXPANSION",
     S"PARAMETER ARE: ", //, 25X, "Y1=", G11.5, 13X, "YO=", 811.5, 13X,
     S"EPS=", G11.5, /)
 913  FORMAT(20X, "THE VALUE OF THE NOZZLE DISCHARGE COEFFICIENT",
     S"IS: ", //, 25X, "CD=", G11.5)
 914  FORMAT(1H1, //, 42X, "DATA FOR A CONTOUR OF CONSTANT ", A8,
     S"(NVAR=", 11, "-", //, 51X, "VALUE=", 811.5, SX, "NTERM=", 12, /,
     S51X, "NPTS=", I3, 14X, "NSOLVz", I3, //)

```

## TRANNOZ CODE...PROGRAM MAIN (CONT.)

```

915 FORMAT(16X,"Z",13X,"R",13X,"U",13X,"V",13X,"M",10X,"THETA",
$11X,"M",12X,"P/PO",/,(/,8X,8(3X,G11.5)))
916 FORMAT(1H1,/)
917 FORMAT(1H1,/,36X,"SUPERSONIC STARTING LINE DATA--",
$"CONSTANT MACH NUMBER CONTOUR",//,51X,"NPTS=",13,14X,
$"NSDLY=",13,/,51X,"INTERM=",12,/)
918 FORMAT(1H1,/,41X,"FLOWFIELD DATA ALONG A CONSTANT X--",
$"COORDINATE PLANE",//,51X,"X=",G11.5,9X,"NPTS=",13,/,51X,
$"INTERM=",12,/)
919 FORMAT(17X,"Y",15X,"U",15X,"V",15X,"M",12X,"THETA",13X,
$"M",14X,"P/PO",/,(/,7X,7(5X,G11.5)))
920 FORMAT(1H1,/,41X,"FLOWFIELD DATA ALONG A CONSTANT Z--",
$"COORDINATE PLANE",//,51X,"Z=",G11.5,9X,"NPTS=",13,/,51X,
$"INTERM=",12,/)
921 FORMAT(17X,"R",15X,"U",15X,"V",15X,"M",12X,"THETA",13X,
$"M",14X,"P/PO",/,(/,7X,7(5X,G11.5)))
C 170 STOP
END

```

## TRANNOZ CODE...FUNCTION IBND

```
REAL FUNCTION IBND(Z)
C...FUNCTION IBND(Z) IS THE EQUATION OF THE INNER WALL CONTOUR
C...IN CYLINDRICAL COORDINATES IN THE FORM R=IBND(Z). IF
C...NGEOM=2, THE INNER BOUNDARY IS A STRAIGHT LINE IN THE MERIDI-
C...NAL PLANE. OTHERWISE, IT IS A CIRCULAR ARC.
C
COMMON/BULKIRC/NGEOM,A1,B1,RC1,A0,BO,RC0
IF(NGEOM.EQ.2) GO TO 10
IBND=B1+SQRT(RC1*RC1-(Z-A1)*(Z-A1))
RETURN
10 IBND=A1*Z+B1
RETURN
END
```

## TRANNOZ CODE...FUNCTION OBND

```

FUNCTION OBND(Z)
C
C...FUNCTION OBND(Z) IS THE EQUATION OF THE OUTER WALL CONTOUR
C...IN CYLINDRICAL COORDINATES IN THE FORM R=OBND(Z). IF
C...NGEOM=3, THE OUTER BOUNDARY IS A STRAIGHT LINE IN THE MERIDI-
C...NAL PLANE. OTHERWISE, IT IS A CIRCULAR ARC.
C
COMMON/BLKCIRC/NGEOM,A1,B1,RC1,A0,B0,RC0
IF(NGEOM.EQ.3) GO TO 10
OBND=B0-SQRT(RC0*RC0-(Z-A0)*(Z-A0))
RETURN
OBND=A0*Z+B0
RETURN
END
10

```

```

CBN 10
CBN 20
CBN 30
CBN 40
CBN 50
CBN 60
CBN 70
CBN 80
CBN 90
CBN 100
CBN 110
CBN 120
CBN 130
CBN 140

```

TRANNOZ CODE... SUBROUTINE ARMIN

```

SUBROUTINE ARMIN
C   . . . SUBROUTINE ARMIN FINDS THE MINIMUM AREA CROSS-SECTION FOR
C   . . . ANNUAL AXISYMETRIC, SUPERSONIC NOZZLES GIVEN THE EQUATIONS
C   . . . FOR THE INNER AND OUTER BOUNDARIES, R=IBND(Z) AND R=OBND(Z),
C   . . . RESPECTIVELY.  ONCE THE MINIMUM AREA SECTION HAS BEEN FOUND,
C   . . . EVALUATION OF SOME INITIAL PARAMETERS IN THE X-Y COORDINATES
C   . . . OF THE TRANSONIC ANALYSIS ARE CARRIED OUT.
C
      REAL IBND
      COMMON/BLKIN/G,ETA,ZIMIN,ZIMAX,ZOMIN,ZOMAX,BLKGEOM/R1,
     $Z1,RO,ZO,ASTAR,D,ZSTAR,BETA,HIP,G1P,H2P,G2P/BLKPARM/Y1,Y0,EPS,
     $H1,G1,H2,G2,BETA
      DATA PI,DX,DY,TOLER/3.1415926535898,0.001,1.0E-10/
     FAREA(R1,Z1,RO,ZO)=PI*(RO+R1)*SQR((Z0-Z1)**2+(RO-R1)**2)
     CONV(OLD,NEW)=ABS((NEW-OLD)/NEW)
C
C   . . . EVALUATE SOME INITIAL CONSTANTS AND PARAMETERS
C
      Z1=ZIMIN    S  Z0=ZOMIN
      RI=IBND(Z1)  S  RO=OBND(Z0)
      ARIM1=FAREA(R1,Z1,RO,ZO)

C   . . . FIND THE MINIMUM AREA SECTION BY ALTERNATELY PIVOTING ON POINTS
C   . . . ON THE UPPER AND LOWER BOUNDARIES
C
      DO 170 I=1,200
      IF(2*(I/2)-1) 10,20,20
      10  ZL=ZIMIN    S  ZR=ZIMAX
      20  ZL=ZOMIN    S  ZR=ZOMAX
      30  DZ=120 J=1,20
          DZ=(ZR-ZL)/20.
      DO 100 K=1,21
      ZK=ZL+FLOAT(K-1)*DZ
      IF(2*(I/2)-1) 40,50,50
      40  ARK=FAREA(IBND(ZK),ZK,RO,ZO)  S  GO TO 60
      50  ARK=FAREA(RO,Z1,OBND(ZK),ZK)
      60  IF(K-2) 90,80,70
      70  IF(ARK .GT. ARKM1) GO TO 110
      80  ZKM2=ZKM1  S  ARKM2=ARKM1
      90  ZKM1=ZK      S  ARKM1=ARK
      100 CONTINUE
      110 IF(I .NE. 1) CALL ERROR("ARMIN",1)
      120 IF(AMAX1(CONV(ARKM2,ARKM1),CONV(ARKM1,ARK),CONV(ARKM2,ARK)) .LT.
           TOLER/10.) GO TO 130
           ZL=ZK/2  S  ZR=ZK
      130 CALL ERROR("ARMIN",2)
      140 IF(2*(I/2)-1) 140,150,150
      150 Z1=ZKM1  S  RI=IBND(Z1)  S  GO TO 160
      160
      170 CONTINUE

```

## TRANNOZ CODE . . SUBROUTINE ARMIN (CONT.)

```

150  ZO=2KM1      $    RO=OBND(ZO)          ARM 510
160  ARI=AR(M1)   $    IF((2*(1/2)) .EQ. 1) .AND. (CONV(ARI,AR(M1)) .LT. TOLER) GO TO 180  ARM 520
     AR(M1)=ARI
170  CONTINUE
C   CALL ERROR("ARMIN",3)
C   . . . THE MINIMUM AREA CROSS SECTION HAS BEEN FOUND. EVALUATE SOME
C   . . . INITIAL PARAMETERS FOR THE TRANSONIC ANALYSIS.
C
180  ASTAR=ARI
     D=SQRT((RO-RI)**2+((ZO-Z1)**2)
2STAR=ZO-RO*(ZO-Z1)/(RO-RI)
     IF(ABS(ARI)-1.0E-4) 190,190,200
190  BETA=0.        $    GO TO 210
200  BETA=ATAN((ZSTAR-Z1)/RI)
     CALL TRRZXY(RI,Z1,X1,Y1)
210  CALL TRRZXY(RO,Z0,X0,Y0)
C   . . . ITERATE FOR THE VALUES Y=H(-DX), H(+DX), G(-DX), G(+DX) SO
C   . . . THAT H'(0), H'(0), G'(0), G'(0) CAN BE DETERMINED. SUBROUTINE
C   . . . ITER IS USED.
C
     DO 340 M=1,4
     NI=1      $    NTYPE=1
     GO TO(220,230,240,250),M
220  XITER=-DX   $    YITER=Y0-5.*DY   $    GO TO 260
230  XITER=+DX   $    YITER=Y0-5.*DY   $    GO TO 260
240  XITER=-DX   $    YITER=Y1-5.*DY   $    GO TO 260
250  XITER=+DX   $    YITER=Y1-5.*DY   $    GO TO 260
260  CALL TRXYRZ(XITER,YITER,RITER,ZITER)
     IF(M>2)270,270,280
270  DEP=RITER-OBND(ZITER)   $    GO TO 290
280  DEP=RITER-IBND(ZITER)
290  CALL ITER(YITER,DY,TOLER,+1.0,DEP,0,0,TOLER,NIT,NTYPE,
     $XNEG,YNEG,XPOS,NSIGN1,NSIGN2)
     IF((NIT.GT.11) .AND. (NTYPE.EQ.1)) .OR. (NIT.GT.100) CALL
     $ERROR("ARMIN",4)
     IF(NTYPE .NE. 3) GO TO 280
     GO TO(300,310,320,330),M
300  YOKM1=YITER   $    GO TO 340
310  YOKP1=YITER   $    GO TO 340
320  YIMM1=YITER   $    GO TO 340
330  YIMP1=YITER
340  CONTINUE
C   . . . WITH THESE VALUES OF H AND G DETERMINED, EVALUATE THE REMAINING
C   . . . PARAMETERS. THE DERIVATIVES ARE APPROXIMATED AS SECOND ORDER
C   . . . FINITE DIFFERENCES.
C

```

## TRANNOZ CODE... SUBROUTINE ARMIN (CONT.)

```
H1P=(Y0XP1-Y0XM1)/(2.*DX)      S   G1P=(Y1XP1-Y1XM1)/(2.*DX)
H2P=(Y0XM1-2.*Y0*Y0XP1)/(DX**2) .
G2P=(Y1XM1-2.*Y1*Y1XP1)/(DX**2)
EPS=(H2P-G2P)/(2.*ETA*(H2P-G2P))
H1=H1P/(SQRT((G+1.)/2.)*EPS**1.5)
G1=G1P/(SQRT((G+1.)/2.)*EPS**1.5)
H2=2.*H2P/(H2P-G2P)      S   G2=2.*G2P/(H2P-G2P)
BETA1=TAN(BETA)/(SQRT((G+1.)/2.)*EPS**1.5)
RETURN
END
```

TRANNOZ CODE... SUBROUTINE DISCO

```
SUBROUTINE DISCO(INTERM, FLOWCO)
C   .. SUBROUTINE DISCO CALCULATES THE DISCHARGE (OR FLOW) COEFFICIENT,
C   .. FLOWCO, TO NTERM TERMS FOR A GIVEN NOZZLE CONFIGURATION.
C
REAL MSTAR,M
COMMON/BLKDEFV/U,V,MSTAR,THETA,M,PPO,CD
CALL AATRANS(0.0,0.0,INTERM,.T.)
FLOWCO=CD
RETURN
END
```

DIS 10  
DIS 20  
DIS 30  
DIS 40  
DIS 50  
DIS 60  
DIS 70  
DIS 80  
DIS 90  
DIS 100  
DIS 110

TRANNOZ CODE . . . SUBROUTINE CONTOUR

```

      SUBROUTINE CONTOUR(NVAR, YVALUE, NPTS, NTERM)
C... SUBROUTINE CONTOUR FINDS THE R-Z COORDINATES OF A MAXIMUM OF
C... 53 POINTS ON THE CONSTANT M, MSTAR, OR P/PO CONTOURS IN THE
C... TRANSonic REGION WINDOWED BY (ZIMIN, ZIMAX) AND (ZOMIN, ZOMAX)
C... ON THE INNER AND OUTER BOUNDARIES, RESPECTIVELY. NPTS IS THE
C... NUMBER OF POINTS REQUESTED ON THE CONTOUR AND NSOLV IS THE
C... NUMBER ACTUALLY FOUND. SINCE THE CONTOUR MAY PASS OUT OF THE
C... WINDOW AREA, IF NVAR=1, THE DEPENDENT VARIABLE IS MACH NUMBER,
C... FOR NVAR=2 IT IS MSTAR, AND FOR NVAR=3 IT IS P/PO. VALUE IS
C... THE VALUE OF THE DEPENDENT VARIABLE AND NTERM IS THE NUMBER OF
C... TERMS FROM THE EXPANSION SOLUTION TO BE INCLUDED. THE CONTOUR
C... INFORMATION IS STORED IN ARRAYS RCONT-NPOCONT.

      REAL IBND, IVAR, MSTAR, M, MCONT, MCOUNT
      COMMON/BLKIN/G,ETA,ZIMIN,ZIMAX,ZOMIN,ZOMAX/BLKPARM/Y1,Y0,
      SEFS,H1,G1,H2,G2,BETA/BLKCOND/NSOLV,RCONT(53),ZCONT(53),
      SVCONT(53),MCOUNT(53),THCONT(53),PPOCANT(53)/BLKDEPV/U,
      SV,MSTAR,THETA,M,PPO,CD
      C
      C... SET INITIAL VALUES
      C
      NSOLV=0      S   DY=(Y0-Y1)/FLOAT(NPTS-3)  S   YIM1=x2.*Y0
      CALL TRRZXY(CBND(ZIMIN),ZIMIN,XIMIN,YIMIN)
      CALL TRRZXY(CBND(ZIMAX),ZIMAX,XIMAX,YIMAX)
      CALL TRRZXY(CBND(ZOMIN),ZOMIN,XOMIN,YOMIN)
      CALL TRRZXY(CBND(ZOMAX),ZOMAX,XOMAX,YOMAX)

      C
      C... SET UP DO LOOP TO FIND POINTS ON CONTOURS
      C
      DO 90 I=1,NPTS
      IF(I .EQ. 1) GO TO 10
      IF(I .EQ. NPTS) GO TO 20
      YITER=Y0-FLGAT(I-2)*DY
      IVAR=XITER=XOMIN+(XIMIN-XOMIN)/(YIMIN-YOMIN)*(YITER-YOMIN)
      XMAX=XOMAX+(XIMAX-XOMAX)/(YIMAX-YOMAX)*(YITER-YOMAX)
      DIVAR=(XMAX-XITER)/20.
      GO TO 30
10     IVAR=ZOMIN  S   DIVAR=(ZOMAX-ZOMIN)/20.
      GO TO 30
20     IVAR=ZIMIN  S   DIVAR=(ZIMAX-ZIMIN)/20.

      C... INITIALIZE QUANTITIES FOR AND CALL SUBROUTINE ITER
      C
      30   NIT=1      S   NTYPE=1
      IF(I .EQ. 1) CALL TRRZXY(CBND(IVAR),IVAR,XITER,YITER)
      IF(I .EQ. NPTS) CALL TRRZXY(CBND(IVAR),IVAR,XITER,YITER)
      IF(I .NE. 1 .AND. I .NE. NPTS) XITER=IVAR
      CALL AATRANS(XITER,YITER,NTERM,.F.)
      CALL VARSOR(NVAR,DEP)

```

## TRANNOZ CODE... SUBROUTINE CONTOUR (CONT.)

```

DVAR=(DEP-VALUE)/VALUE
CALL ITER(IVAR,DIVAR,1.0E-6,.1,DVAR,0.0,1.0E-6,NIT,NTYPE,
SXNEG,YNEG,XPOS,YPOS,NSIGN1,NSIGN2)
IF(NIT.GT.21) AND. NTYP(.EQ.1) GO TO 30
IF(NIT.GT.100) CALL ERROR('CONTOUR',5)
IF(NTYPE.NE.3) GO TO 40
C.. THE SOLUTION POINT HAS BEEN FOUND. CHECK TO MAKE SURE THAT THE
C.. POINT IS WITHIN THE REGION OF INTEREST, AND IF SO, STORE THE
C.. R-Z COORDINATES OF THE POINT AND VARIOUS PROPERTIES (U,V,RH,
C.. THETA,M,P/PO) IN CORRESPONDING ARRAYS.
C
IF(I .NE. 1) GO TO 50
RSOLN=OBND(IVAR)      S ZSOLN=IVAR
CALL TRRZAY(RSOLN,ZSOLN,XITER,YITER)
GO TO 70
IF(I .NE. NPTS) GO TO 60
RSOLN=IBND(IVAR)      S ZSOLN=IVAR
CALL TRRZXY(RSOLN,ZSOLN,XITER,YITER)
IF(ABS(YITER-YIM1) .LE. 0.001) GO TO 80
GO TO 70
XITER=IVAR
CALL TRXYRZ(XITER,YITER,RSOLN,ZSOLN)
IF(ABS(YITER-YIM1) .LE. 0.001) GO TO 90
IF(I .GT. 3) GO TO 65
IF(RSOLN .GT. OBND(ZSOLN)) GO TO 90
IF(I .LT. NPTS-2) GO TO 70
IF(RSOLN .LT. IBND(ZSOLN)) GO TO 90
YIM1=YITER      S NSOLV=NSOLV+1
RCONT(NSOLV)=RSOLN      S ZCONT(NSOLV)=ZSOLN
CALL AATRANS(XITER,YITER,INTERM,.F.)
UCONT(NSOLV)=U      S YCONT(NSOLV)=V
MSCONT(NSOLV)=MSTAR      S THCONT(NSOLV)=THETA
MCOUNT(NSOLV)=M      S PPOCOUNT(NSOLV)=PPPO
CONTINUE
RETURN
END

```

TRANNOZ CODE... SUBROUTINE STLINE

```

SUBROUTINE STLINE(NPTS,NTERM)

C   .. SUBROUTINE STLINE CALCULATES A SUPERSONIC INITIAL VALUE LINE
C   .. FOR USE IN STARTING METHOD OF CHARACTERISTICS (OR SIMILAR)
C   .. CALCULATIONS FOR ANNULAR SUPERSONIC NOZZLES. THE CONSTANT
C   .. MACH NUMBER LINE FROM THE THROAT WALL LOCATION WITH THE HIGHER
C   .. MACH NUMBER IS USED. NPTS IS THE NUMBER OF POINTS ON THE
C   .. STARTING LINE (MAXIMUM=53) AND NTERM SPECIFIES THE NUMBER OF
C   .. TERMS FROM THE ANALYSIS TO BE USED.

REAL MSTART,MSTAR,M,MSCONT,MCONT
COMMON/BLKPARM/Y1,Y0,EPS,H1,G1,H2,G2,BETA1/BLKDEPV/U,V,MSTAR,
STHETA,M,PPO,CD/BLKCGNT/NSOLV,RCONT(53),ZCONT(53),UCONT(53),
SVCNT(53),MSCONT(53),MCOUNT(53),TCOUNT(53),PCOUNT(53)

C   .. CHECK THE MACH NUMBERS AT THE INNER AND OUTER THROAT WALL
C   .. LOCATIONS
C   CALL AATRANS(0.0,Y1,NTERM,.F.)
MSTART=M
CALL AATRANS(0.0,Y0,NTERM,.F.)
IF(M .GT. MSTART) RSTART=M
IF(MSTART .LE. 1.0) CALL ERROR("STLINE",6)

C   .. GENERATE THE INITIAL VALUE LINE. COORDINATES AND CORRE-
C   .. SPONDING FLOW PROPERTIES ALONG THE LINE ARE STORED IN THE
C   .. ARRAYS RCONT-PCOUNT.

NTEMP=NPTS+1
DO 10 I=1,2
    CALL CONTOUR(1,MSTART,NTEMP,NTERM)
    IF(NSOLV .EQ. NPTS) RETURN
    NTEMP=NTEMP+NPTS-NSOLV
    RETURN
10 END

```

TRANNOZ CODE... SUBROUTINE XPLANE

```

SUBROUTINE XPLANE(XPL,NPTS,NTERM)

C... SUBROUTINE XPLANE EVALUATES THE DEPENDENT VARIABLES U,V,Mx,
C... THETA, M, P/PO AND PERTURBATION VELOCITY COMPONENTS U1,V1,U2,
C... V2,U3,V3 AT NPTS POINTS FROM Y1 TO Y0 ALONG THE PLANE X=XPL
C... IN THE NOZZLE THRUST. A MAXIMUM OF NPTS=51 POINTS IS ALLOWED.
C... NTERM TERMS IN THE SERIES SOLUTION ARE USED. THE RESULTS ARE
C... RETURNED IN ARRAYS XPL,V3XPL.

C      REAL MSXPL,MXPL,MSTAR,M
C      COMMON /BLKPARM/Y1,Y0,EPS,H1,G1,H2,G2,BETA1/BLKXPL/YXPL(51),
C      SUXPL(51),VXPL(51),MSXPL(51),THXPL(51),MXPL(51),PP0XPL(51),
C      SU1XPL(51),V1XPL(51),U2XPL(51),V2XPL(51),U3XPL(51),V3XPL(51)
C      S/BLKDEPV/U,V,MSTAR,THETA,M,PP0,CD/BLKCMP/U1,V1,U2,V2,U3,V3
C      XPL(90) 100
C      XPL(90) 200
C      XPL(90) 300
C      XPL(90) 400
C      XPL(90) 500
C      XPL(90) 600
C      XPL(90) 700
C      XPL(90) 800
C      XPL(90) 900
C      XPL(100) 100
C      XPL(110) 110
C      XPL(120) 120
C      XPL(130) 130
C      XPL(140) 140
C      XPL(150) 150
C      XPL(160) 160
C      XPL(170) 170
C      XPL(180) 180
C      XPL(190) 190
C      XPL(200) 200
C      XPL(210) 210
C      XPL(220) 220
C      XPL(230) 230
C      XPL(240) 240
C      XPL(250) 250
C      XPL(260) 260
C      XPL(270) 270
C      XPL(280) 280
C      XPL(290) 290
C      XPL(300) 300
C      XPL(310) 310
C      XPL(320) 320

C... CALL AATRANS TO EVALUATE THE VARIOUS QUANTITIES:
C
C      DY=(Y0-Y1)/FLOAT(NPTS-1)
C      DO 10 I=1,NPTS
C          YXPL(I)=Y1+FLOAT(I-1)*DY
C          CALL AATRANS(XPL,YXPL(I),NTERM,.F.)
C          UXPL(I)=U   $  VXPL(I)=V
C          MSXPL(I)=MSTAR $  THXPL(I)=THETA
C          MXPL(I)=M   $  PP0XPL(I)=PP0
C          U1XPL(I)=U1 $  V1XPL(I)=V1
C          IF(NTERM.EQ. 1) GO TO 10
C          U2XPL(I)=U2 $  V2XPL(I)=V2
C          IF(NTERM.EQ. 2) GO TO 10
C          U3XPL(I)=U3 $  V3XPL(I)=V3
C      CONTINUE
C      RETURN
C      END
10

```

TRANNOZ CODE . . . SUBROUTINE ZPLANE

```

SUBROUTINE ZPLANE(ZPL,NPTS,NTERM)
C
C... SUBROUTINE ZPLANE EVALUATES THE DEPENDENT VARIABLES U,V,M,
C... THETA, M, P/PO AND PERTURBATION VELOCITY COMPONENTS U1,V1,U2,
C... V2,U3,V3 AT NPTS POINTS FROM THE INNER TO THE OUTER WALL
C... ALONG THE PLANE Z=ZPL (PERPENDICULAR TO THE AXIS OF SYMMETRY).
C... A MAXIMUM OF NPTS=51 POINTS IS ALLOWED. NTERM TERMS IN THE
C... SERIES SOLUTION ARE USED. THE RESULTS ARE RETURNED IN ARRAYS
C... RZPL-V3ZPL.
C
      REAL IBND,MZPL,MZPL,MSTAR,M
      COMMON/BLKZPL/RZPL(51),UZPL(51),VZPL(51),THZPL(51),
     $MZPL(51),PP0ZPL(51),U1ZPL(51),V1ZPL(51),U2ZPL(51),
     $U3ZPL(51),V3ZPL(51)/BLKDEPV/U,V,MSTAR,THETA,M,PP0,CD/BLKCOMP/
     $U1,V1,U2,V2,U3,V3
C... CALL AATRANS TO EVALUATE THE VARIOUS QUANTITIES:
C
      RIN=IBND(ZPL)           S   ROUT=OBND(ZPL)
      DR=(ROUT-RIN)/FLOAT(NPTS-1)
      DO 10 I=1,NPTS
      RZPL(I)=RIN+FLOAT(I-1)*DR
      CALL TRRZXY(RZPL(I),ZPL,X,Y)
      CALL AATRANS(X,Y,NTERM,F.)
      UZPL(I)=U           S   VZPL(I)=V
      MZPL(I)=MSTAR       S   THZPL(I)=THETA
      MZPL(I)=M           S   PP0ZPL(I)=PP0
      U1ZPL(I)=U1          S   V1ZPL(I)=V1
      IF(NTERM.EQ. 1) GO TO 10
      U2ZPL(I)=U2          S   V2ZPL(I)=V2
      IF(NTERM.EQ. 2) GO TO 10
      U3ZPL(I)=U3          S   V3ZPL(I)=V3
      CONTINUE
      RETURN
      END
10

```

TRANSDZ CODE . . . SUBROUTINE TRRZXY

C SUBROUTINE TRRZXY CARRIES OUT THE TRANSFORMATION FOR A POINT  
C . . . WITH CYLINDRICAL COORDINATES (R,Z) TO THE (X,Y) COORDINATES  
C . . . OF THE TRANSONIC ANALYSIS. D IS THE NON-DIMENSIONALIZING  
C . . . DISTANCE, ZSTAR IS THE DISPLACEMENT OF THE X-Y ORIGIN FROM  
C . . . THE R-Z ORIGIN, AND BETA IS THE ANGLE OF INCLINATION OF THE  
C . . . X-AXIS WITH RESPECT TO THE Z-AXIS.  
C  
COMMON/BLKGEOM/R1,Z1,R0,Z0,ASTAR,D,ZSTAR,BETA,HIP,H2P,H2Z  
X=(Z-ZSTAR)/D\*COS(BETA)+R/D\*SIN(BETA)  
Y=(Z-ZSTAR)/D\*SIN(BETA)+R/D\*COS(BETA)  
RETURN  
END

TRANNOZ CODE . . . SUBROUTINE TRXYRZ

```
      SUBROUTINE TRXYRZ(X,Y,R,Z)
C
C   . . . SUBROUTINE TRXYRZ CARRIES OUT THE TRANSFORMATION FOR A POINT
C   . . . WITH COORDINATES (X,Y) OF THE TRANSONIC ANALYSIS TO CYLINDRI-
C   . . . CAL COORDINATES (R,Z).  D IS THE NON-DIMENSIONALIZING DISTANCE,
C   . . . ZSTAR IS THE DISPLACEMENT OF THE X-Y ORIGIN FROM THE R-Z ORIGIN,
C   . . . AND BETA IS THE ANGLE OF INCLINATION OF THE X-AXIS WITH RESPECT
C   . . . TO THE Z-AXIS.
C
C   COMMON/BLKGEOM/R1,Z1,RO,Z0,ASTAR,D,ZSTAR,BETA,H1P,G1P,H2P,G2P
C   R=X*D*SIN(BETA)+Y*D*COS(BETA)
C   Z=ZSTAR+X*D*COS(BETA)-Y*D*SIN(BETA)
C   RETURN
C   END
```

TRANNOZ CODE... SUBROUTINE ITER

SUBROUTINE ITER(X,DX,ERRORX,SIGN,Y,YGIVEN,ERRORY,NIT,NTYPE,  
SXNEG,YNEG,XPOS,YPOS,NSIGN1,NSIGN2)

C... SUBROUTINE ITER PERFORMS AN ITERATION TO FIND X SUCH THAT THE  
C... ABSOLUTE VALUE OF (Y-YGIVEN) IS LESS THAN OR EQUAL TO ERRORY  
C... OR THE ABSOLUTE VALUE OF (X(1+1)-X(1)) IS LESS THAN OR EQUAL  
C... TO ERRORX.

C... VARIABLES:

```

C   X      = INDEPENDENT VARIABLE
C   DX     = INCREMENT IN INDEPENDENT VARIABLE
C   ERRORX = MAXIMUM VALUE OF ABS(X(1+1)-X(1)) FOR SOLN
C   SIGN   = +1.0 OR -1.0, DEFINES INCREMENTING FROM X INITIAL
C   Y      = DEPENDENT VARIABLE
C   YGIVEN = GIVEN VALUE OF DEPENDENT VARIABLE
C   ERRORY = MAXIMUM VALUE OF ABS(Y-YGIVEN)
C   NIT    = INCREMENT NUMBER
C   NTYPE  = 1--INCREMENT, 2--INTERPOLATION, 3--SOLUTION

C   DY=Y-YGIVEN
C   IF(Abs(DY)=ERRORY) 90,90,10
C   IF(DY) 20,90,30

10   NSIGN2=-1
C   XNEG=X   S   YNEG=Y
C   GO TO 40

C   30   NSIGN2=+1
C   XPOS=X   S   YPOS=Y
C   40   IF(NTYPE .EQ. 2) GO TO 60
C   50   IF(NIT-1) 70,70,60
C   60   NSIGN=NSIGN1*NSIGN2
C   70   IF(NSIGN) 80,80,70
C   80   NSIGN1=NSIGN2   S   NIT=NIT+1
C   90   NSIGN=NSIGN2   S   NIT=NIT+1
C   100  RETURN
C   END

C... INCREMENT TO FIND SOLUTION INTERVAL
C   X=X+SIGN*DX
C   GO TO 100

C... INTERPOLATION FOR SOLUTION
C   80   NTYPE=2   S   NIT=NIT+
C   XSAVE=X   S   RATIO=(XPOS-XNEG)/(YPOS-YNEG)
C   X=XNEG+RATIO*(YGIVEN-YNEG)
C   IF(ABS(X-XSAVE)>ERRORX) 90,90,100
C   90   NTYPE=3
C   100  RETURN

```

TRANNOZ CODE . . . SUBROUTINE VARSOR

```
SUBROUTINE VARSOR(NVAR,DEP)

C... SUBROUTINE VARSOR PUTS THE VALUE OF M, MSTAR, OR P/PO INTO
C... DEP DEPENDING ON THE VALUE OF NVAR.
C... FOR:
C...   NVAR=1,      DEP=M
C...   NVAR=2,      DEP=MSTAR
C...   NVAR=3,      DEP=P/PO
C... .
C... REAL MSTAR,M
COMMON/BLKDEPV/U,V,MSTAR,THETA,M,PPO,CD
GO TO(10,20,30),NVAR
10 DEP=M      S RETURN
20 DEP=MSTAR  S RETURN
30 DEP=PPO   S RETURN
END
```

## TRANNOZ CODE ... SUBROUTINE ERROR

```

SUBROUTINE ERROR(ROUT, IER)
C... SUBROUTINE ERROR WRITES DIAGNOSTIC MESSAGES FOR ERROR CONDITIONS ENCOUNTERED IN OTHER SUBROUTINES. ROUT NAMES THE SUBROUTINE WHERE THE ERROR OCCURRED AND IER IS THE ERROR NUMBER.
C... FORMAT STATEMENTS
C
      WRITE(6,900) ROUT
      60 TG(10,20,30,40,50,60),IER
      10 WRITE(6,901) IER  $ 80 TG 100
      20 WRITE(6,902) IER  $ 80 TG 100
      30 WRITE(6,903) IER  $ 80 TG 100
      40 WRITE(6,904) IER  $ 80 TG 100
      50 WRITE(6,905) IER  $ 80 TG 100
      60 WRITE(6,906) IER
      100 WRITE(6,910)

C
      FORMAT(1H1, //,27X,"AN ERROR OCCURRED WHILE COMPUTATIONS ")
      900 S"WERE BEING CARRIED OUT IN SUBROUTINE ",A7, //,52X,"THE "
      S"DIAGNOSIS OF THE PROBLEM IS: " //)
      901 FORMAT(28X,"ERROR NO. -12,- : AREA NOT FOUND TO INCREASE "
      S"IN INNER LOOP OF MINIMIZING ITERATIONS")
      902 FORMAT(19X,"ERROR NO. -12,- : CONVERGENCE FOR MINIMUM AREA "
      S"WITH A GIVEN PIVOT POINT DID NOT OCCUR IN 20 ITERATIONS")
      903 FORMAT(14X,"ERROR NO. -12,- : CONVERGENCE FOR MINIMUM AREA "
      S"NOT OBTAINED FOR 100 PIVOTS EACH ON THE UPPER AND LOWER "
      S"BOUNDARIES")
      904 FORMAT(27X,"ERROR NO. -12,- : ITERATIONS FOR H(-DX),H(+DX) "
      S",G(-DX), OR G(+DX) DID NOT CONVERGE")
      905 FORMAT(19X,"ERROR NO. -12,- : ITERATIONS FOR COORDINATES OF "
      S"CONSTANT H,M, OR P/PO CONTOUR POINTS DID NOT CONVERGE")
      906 FORMAT(35X,"ERROR NO. -12,- : MACH NUMBER FOR INITIAL VALUE "
      S"LINE IS NOT SUPERSONIC")
      910 FORMAT(//,58X,"EXECUTION TERMINATED")

END

```

TRANNO2 CODE... SUBROUTINE AATRANS

```

C          SUBROUTINE AATRANS(XS,YS,NTERM,FCOEF)
C
C... SUBROUTINE AATRANS COMPUTES THE TRANSONIC FLOWFIELD IN THE
C... THROAT REGION OF INCLINED, ANULAR AXISYMMETRIC, SUPERSONIC
C... NOZZLES. THE METHOD USED IS EVALUATION OF A PERTURBATION
C... SERIES SOLUTION SIMILAR TO THOSE OF HALL AND THOMPSON AND
C... FLACK. THE QUANTITIES RETURNED ARE U,V,M, ,THETA,M,P/PO, AND
C... CD
C
C      IMPLICIT DOUBLE PRECISION(A-H,M,O-Z)
C
C      LOGICAL FCOEF
C      REAL XS,YS,GS,ETAS,ZIMIN,ZIMAX,ZOMIN,ZOMAX,Y1S,Y0S,EPSS,
C      SH1S,G1S,H2S,G2S,BETA1S,US,VS,MSTARS,THETAS,MS,PPOS,CDS,U1S,
C      SV1S,U2S,V2S,U3S,V3S
C      COMMON/BLKIN/GS,ETAS,ZIMIN,ZIMAX,ZOMIN,ZOMAX/BLKPARM/
C      SY1S,Y0S,EPSS,H1S,G1S,H2S,G2S,BETA1S/BLKDEPV/US,VS,MSTARS,
C      SHETAS,MS,PPOS,CDS/BLKCOMP/U1S,V1S,U2S,V2S,U3S,V3S
C      S/BLKCALL/ICALL1,ICALL2,ICALL3
C
C... CALCULATE SOME INITIAL CONSTANTS:
C
C      ICALL1=ICALL1+1
C      IF(ICALL1.GT.1) GO TO 10
C      ETA=DBLE(ETAS)   S  EPS=DBLE(EPSS)   S  Y1=DBLE(Y1S)
C      Y0=DBLE(Y0S)   S  G1=DBLE(G1S)   S  G2=DBLE(G2S)
C      H1=DBLE(H1S)   S  H2=DBLE(H2S)   S  BETAI=DBLE(BETA1S)
C      G=DBLE(GS)   S  GP1=G+1.   S  GM1=G-1.
C      CWT=DSQRT(GP1*.5*EPS)
C      CM=GP1*.5   S  CPP0=(2./GP1)**(G/GM1)
C      CCD=GP1*EPS*EPS/(Y0*Y0-Y1*Y1)   S  CDS=1.0
C      U2S=0.0   S  V2S=0.0
C      U3S=0.0   S  V3S=0.0
C
C... CALCULATE THE REQUIRED QUANTITIES FOR THE FIRST ORDER SOLUTION.
C... FIRST THE VARIOUS YI AND YO CONSTANTS:
C
C      YOE2=Y0*YO   S  YIE2=Y1*YI
C      YOE3=Y0E2*Y0   S  YIE3=Y1E2*YI
C      YOE4=Y0E2*YOE2   S  YIE4=Y1E2*YIE2
C      YOE5=Y0E2*YOE3   S  YIE5=Y1E2*YIE3
C      YOE6=Y0E3*YOE3   S  YIE6=Y1E3*YIE3
C      YOC0=DLGG(YO)   S  YICO=DLGG(YI)
C      YOC1=Y0*YOC0   S  YIC1=Y1*YICO
C      YOC2=YOC1*YOC1   S  YIC2=Y1C1*YIC1
C      YOC3=Y0E4*YOC0   S  YIC3=Y1E4*YICO
C      YOC4=Y0E2*YOC0   S  YIC4=Y1E2*YICO
C
C... CALCULATE THE "B" CONSTANTS:
C
C      B2=-(H2*Y1-G2*Y0)*Y0*Y1/(Y0E2-YIE2)   S  B2E2=B2*Y2
C
C      10          AAT 10
C      20          AAT 20
C      30          AAT 30
C      40          AAT 40
C      50          AAT 50
C      60          AAT 60
C      70          AAT 70
C      80          AAT 80
C      90          AAT 90
C     100         AAT 100
C     110         AAT 110
C     120         AAT 120
C     130         AAT 130
C     140         AAT 140
C     150         AAT 150
C     160         AAT 160
C     170         AAT 170
C     180         AAT 180
C     190         AAT 190
C     200         AAT 200
C     210         AAT 210
C     220         AAT 220
C     230         AAT 230
C     240         AAT 240
C     250         AAT 250
C     260         AAT 260
C     270         AAT 270
C     280         AAT 280
C     290         AAT 290
C     300         AAT 300
C     310         AAT 310
C     320         AAT 320
C     330         AAT 330
C     340         AAT 340
C     350         AAT 350
C     360         AAT 360
C     370         AAT 370
C     380         AAT 380
C     390         AAT 390
C     400         AAT 400
C     410         AAT 410
C     420         AAT 420
C     430         AAT 430
C     440         AAT 440
C     450         AAT 450
C     460         AAT 460
C     470         AAT 470
C     480         AAT 480
C     490         AAT 490
C     500         AAT 500

```

TRANNO2 CODE... SUBROUTINE AATRANS (CONT.)

```

B1=DSQRT((H2*Y0-B2)/Y0   S   B1E2=B1*B1
B1E3=B1E2*B1   S   B1E4=B1E2*B1E2
B4=B1E3*Y0*B1   S   B2=B2*Y0C1-B1*B2*Y0   S-BETA1
B5=B1E3*Y1E3* 25+B1*B2*Y1C1-B1*B2*Y1   S-BETA1
B3=- (H1*Y1-G1*Y0-B4*Y1+B5*Y0)*Y0*Y1/(Y0E2-Y1E2)
B0=(H1*Y0-B4*Y0-B3)/(B1*Y0E2)   S   BOE2=BO*BO
B6=BO*B1-B1*B2* .5

C... CALCULATE X,Y,Z, THE Y CONSTANTS, AND THE "A" FUNCTIONS:
C
10 IF(FCDEF) GO TO 20
X=DBLE(XS)   S   Y=DBLE(Y$)
Z=X/DSQRT(GP1* .5*EPS)
YE2=Y*Y   S   YE3=YE2*Y
YCO=DLG(Y)   S   YC1=Y*YCO
A1P=B1E2*Y+B2/Y
A1=B1E2*YE2*.5+B2*YCO
ADP=B1E3*YE3*.25+B1*B2*Y*.5-BETA1+BO*B1*Y+B3/Y

C... CALCULATE THE DESIRED QUANTITIES TO FIRST ORDER:
C
U1=A1+BO+B1*Z   S   V1=AOP+A1P*Z
U=1.+U1*EPS   S   V=CVT*V1*EPS
MSTAR=J   S   THETA=V
M=1.+CM*U1*EPS   S   PPO=CPP0*(1.-G*U1*EPS)
U1S=SNGL(U1)   S   V1S=SNGL(V1)
US=SNGL(U)   S   VS=SNGL(V)
MSTAR=SNGL(MSTAR)   S   THETAS=SNGL(THETA)
MS=SNGL(M)   S   PPOS=SNGL(PPO)
GO TO 30

C... IF DESIRED, CALCULATE THE DISCHARGE COEFFICIENT:
C
20 CD4=BO*B1E2*.25-B1E2*B2/16.
CD5=BOE2*.5+B2E2*.25-BO*B2*.5
CD6=BO*B2-B2E2*.5
CD1=B1E4*(Y0E2-Y1E6)/24.+CD4*(Y0E4-Y1E4)+CD5*(Y0E2-Y1E2)+B2E2*.5
S*(YOC2-Y1C2)+B1E2*B2*.25*(YOC3-Y1C3)+CD6*(YOC4-Y1C4)
CD=1.-CCD*CD1
CDS=SNGL(CD)
IF(INTERM .EQ. 1) RETURN

C... CALCULATE THE REQUIRED QUANTITIES FOR THE SECOND ORDER SOLUTION.
C... FIRST THE VARIOUS B, Y1, Y0 CONSTANTS:
C
ICALL2=ICALL2+
IF(ICALL2 .GT. 1) GO TO 40
B1E3=B1E2*B1E3   S   B1E6=B1E3*B1E3
BOE3=BO*BOE2   S   B2E3=B2*B2E2

```

TRANNOZ CODE... SUBROUTINE ATRANS (CONT.)

```

B3E2=B3*B3      S   B6E2=B6*B6
BETA1E2=BETA1*BETA1
YOE7=YOE3*YOE4      S   YIE7=YIE3*YIE4
YOE8=YOE4*YOE4      S   YIE8=YIE4*YIE4
YOC5=YOCO/YO      S   YIC5=Y1*YOC0/YI,
YOC6=Y0*YOCO*YOC0      S   YIC6=Y1*Y1C0*Y1CO
YOC7=YOE3*YOC0      S   YIC7=YIE3*Y1CO
YOC8=YOC4*YOC4      S   YIC8=Y1C4*Y1CA
YOC9=YOE6*YOC0      S   YIC9=YIE6*Y1CO
YOC10=YOC2*YOC0      S   YIC10=Y1C2*Y1CO

C... CALCULATE THE "D" CONSTANTS:
C
D3=B1E2*YOE2*.5+B2*YOC0+B0
D4=H1*D3
D5=H2*ETA+H2*D3+H1*B1
D6=H2*B1
D7=B1E2*YIE2*.5+B2*Y1CO+B0
D8=G1*D7
D9=G2*ETA+G1*B1+G2*D7
D10=G2*B1
D11=(D10*Y0-D8*Y1)*Y0*Y1/(YOE2*YIE2)
D2=(D6*Y0-(2.*G-1.)*5*B1E3*YOE2-D11)/(3.*B1*YOE2)
D12=(2.*G-1.)*5*B1E3+3.*B1*D2
D13=(2.*B1*D12+2.*B1E2*D2+(2.*G+1.)*B1E4
D14=6.*B1*D11+4.*B2*D2+(4.*G-2.)*B1E2*B2
D15=4.*B1E2*B2+4.*B0*D2+(4.*G-2.)*B0*B1E2
D17=D13*YOE3*.25+(D15*.5-D14*.25)*Y0*D14*.5*YOC1+.2.*B2E2*YOC5-
$BETA1*B1
D18=D13*YIE3*.25+(D15*.5-D14*.25)*Y1*D14*.5*Y1C1+.2.*B2E2*Y1C5-
$BETA1*B1
D16=(D9*Y0-D5*Y1-D16*Y0+D17*Y1)*Y1*Y0/(YOE2-YIE2)
D1=(D5*Y0-D17*Y0-D16)/(2.*B1*YOE2)
D19=D15*.5-D14*.25+2.*B1*D1
D20=D19*.5-D14/6.
D21=B1E2*D12+B1*B1E2*D13/6.+{(2.*G+1.)*.25*B1E5
D22=2.*B0*D12+B1E2*D1+(2.*G-1.)*B0*B1E3+.2.*B1*D20+.2.*B1E2*B6+
$B1E3*B2*.5
D23=-4.5*BETA1*B1E2
D24=-2.*BETA1*B2-BETA1*B0
D25=-4.*B2*D11+{(2.*G+1.)*B1*B2E2
D26=2.*B2*D12+2.*B1E2*D11+B1*D14*.5+{(2.*G+1.)*B1E3*B2
D27=4.*B0*D11+2.*B2*D1+{(4.*G-2.)*B0*B1*B2+.2.*B1*B2E2
D28=2.*B0*D1+{(2.*G-1.)*B0E2*B1+.2.*B1E2*B3+.2.*B2*B6
D30=D21*YOE5/6.+{(D22*.25-D26/16.)*YOE3+D23*YOE2/3.+(D25*.25-D27
$*25+D28*.5)*Y0*D25*YOC6*.5+D26*YOC7*.25+(D27*.5-D25*.5)*YOC1-
$BETA1*B2*YOC0/2.*B2*B3*YOC4+BETA1*B2
D31=D21*YIE5/6.+{(D22*.25-D26/16.)*YIE3+D23*YIE2/3.+(D25*.25-D27
$*25+D28*.5)*Y1*D25*Y1C6*.5+D26*Y1C7*.25+(D27*.5-D25*.5)*Y1C1-
$BETA1*B2*Y1CO/2.*B2*B3*Y1C5+D24*BETA1*B2
D29=(D8*Y0-D4*Y1-D31*Y0+D30*Y1)*Y1*Y0/(YOE2-YIE2)

```

## TRANNOZ CODE... SUBROUTINE AATRANS (CONT.)

```

DO=(D4*Y0-D30*Y0-D29)/(B1*Y0E2)
D32=D22*.25-D26/16.
D33=D25*.25-D27*.25+D28*.5+B1*DO
D34=D27*.5-D25*.5
D35=D24+BETA1*B2

C... CALCULATE THE Y CONSTANTS AND THE "C" FUNCTIONS:
C
 40 IF(FCDEF) GO TO 50
    YE4=YE2*YE2 $ YE5=YE2*YE3
    YC3=YCO/Y $ YC4=YCO*YCO
    YC5=YE2*YCO $ YC6=Y*YC4
    YC7=YE3*YCO
    C2P=D12*Y+D11/Y
    C2=D12*YE2*.5+D11*YCO
    C1P=D13*YE3*.25+D19*Y+D16*Y+D14*YC1*.5+.2*B2E2*YC3-BETA1*B1
    C1=D13*YE4/16.+D20*YE1*B1*Y+B2E2*YC4+D14*YC5*.25+D16*YCO
    COP=D21*YE5/6.+D32*YE3+D23*YE2/3.+D33*Y+D29/Y+D25*YC6*.5+D26*
    SYC7*.25+D34*YC1-BETA1*B2*YCO+2.*B2*B3*YC3+D35
C... CALCULATE THE DESIRED QUANTITIES TO SECOND ORDER:
C
    ZE2=Z*Z $ EPSE2=EPS*EPS
    U2=C1+D0+(2.*C2+C1)*Z+D2*ZE2
    V2=COP+C1P*Z+C2P*ZE2
    U=U+U2*EPSE2 $ V=V+CVT*V2*EPSE2
    MSTAR=U $ THETA=THETA+CVT*(V2-U1*V1)*EPSE2
    M=M+CM*(U2+0.75*GM1*U1*U1)*EPSE2
    PPO=PP0-CPPO*G*U2*EPSE2
    U2S=SNGL(U2) $ V2S=SNGL(V2)
    US=SNC1(U) $ VS=SNGL(V)
    MSTAR=SNGL(MSTAR) $ THEtas=SNGL(THETA)
    MS=SNGL(M) $ PPOS=SNGL(PP0)
    GO TO 60

C... IF DESIRED, CALCULATE THE DISCHARGE COEFFICIENT:
C
 50 CD10=B1E3*B6/24.-B1E4*B2/144.
    CD11=B1E3*B3/16.+B6E2/8.-B1*B2*B6/16.+B1E2*B2E2/64.
    CD12=BETA1*B1*B2/9.-BETA1*B6/3.
    CD13=B3*B6*.5+BETA1E2*.25-B1*B2*B3*.25
    CD14=B1*B2*B6*.25-B1E2*B2E2/16.
    CD7A=B1E6/256.* (Y0E8-Y1E8)+CD10*(Y0E6-Y1E6)-BETA1*B1E3/20.* (Y0E5-
    SY1E5)+CD11*(Y0E4-Y1E4)+CD12*(Y0E3-Y1E3)+CD13*(Y0E2-Y1E2)-BETA1*
    SB3*(Y0-Y1)
    CD7B=B1E2*B2E2/8.* (YOC8-Y1C8)+B1E4*B2/24.* (YOC9-Y1C9)+CD14*
    S*(YOC3-Y1C3)-BETA1*B1*B2/3.* (YOC7-Y1C7)+B1*B2*B3*.5*(YOC4-Y1C4)+
    SB3E2*.5*(YOC0-Y1C0)
    CD7=CD7A+CD7B
    CD15=B1E2*D20/6.+B0*D13/48.-B1E2*D14/144.-B2*D13/288.

```

TRANNOZ CODE... SUBROUTINE AATRANS (CONT.)

```

CD16=B1E2*D0* .25+B0*D20* .5+B1E2*B2E2/32. +B2*D14/64. -B1E2*D16/16. -
S82=B2*D20/8. -B0*D14/32.
CD17=2./9.*BETA1*B1*B2-2./3.*BETA1*B0*B1
CD18=B0*D0-0.75*B2E3+B2*D16*.5+B0*B2E2*.5-B0*D0*.5-B0*D16*.5
CD19=B1E2*B2E2*.25+B2*D14/6.
CD20=B2*D16+B0*B2E2-1.5*B2E3
CD21=B1E2*D14/24. +B2*D13/48.
CD22=B1E2*D16*.25+B2*D20*.5+B0*D14/8. -B1E2*B2E2/8. -B2*D14/16.
CD23=1.5*B2E3-.25*D16-B0*B2E2+B2*D0+B0*D16
CD8A=B1E2*D13/128.*(Y0E8-Y1E8)+CD15*(Y0E6-Y1E6)-BETA1*B1E3/5. *
$(Y0E5-Y1E5)+CD16*(Y0E4-Y1E4)+CD17*(Y0E3-Y1E3)+CD18*(Y0E2-Y1E2)
CDCB=B2E3*(Y0C10-Y1C10)+CD19*(Y0C8-Y1C8)+CD20*(Y0C2-Y1C2)+CD21*
$ (Y0C9-Y1C9)+CD22*(Y0C3-Y1C3)-2./3.*BETA1*B1*B2*(Y0C7-Y1C7)+CD23*
$ (Y0C4-Y1C4)
CD8=CDBA+CDBB
CD24=B0*B1E4/8.-B1E4*B2/48.
CD25=.8.*B0E2*B1E2+3./64.*B1E2*B2E2-3./16.*B0*B1E2*x*B2
CD26=B0E3*.5-0.75*B0E2*B2+0.75*B0*B2E2-3./8.*B2E3
CD27=1.5*B0*B2E2-0.75*B2E3
CD28=0.75*B0*B1E2*B2-3./16.*B1E2*B2E2
CD29=0.75*B2E3-1.5*B0*B2E2+1.5*B0E2*B2
CD9A=B1E6/64.* (Y0E8-Y1E8)+CD24*(Y0E6-Y1E6)+CD25*(Y0E4-
3Y1E4)+CD26*(Y0E2-Y1E2)+B2E3*.5*(Y0C10-Y1C10)+3./8.*B1E2*x
3B2E2*(Y0C8-Y1C8)
CD9B=CD27*(Y0C2-Y1C2)+B1E4*B2/8. *(Y0C9-Y1C9)+CD28*(Y0C3-
3Y1C3)+CD29*(Y0C4-Y1C4)
CD9=(2./6-3./3).* (CD9A+CD9B)
CD2=CD7+CD8+CD9
CD=CD-CCD*CD2*EPS
CDS=SMGL(CD)
60   IF (INTERM .EQ. 2) RETURN
C... CALCULATE THE REQUIRED QUANTITIES FOR THE THIRD ORDER SOLUTION.
C... FIRST THE VARIOUS B,D,YI,YO CONSTANTS:
C
ICALL3=ICALL3+1
IF (ICALL3 .GT. 1) GO TO 70
ETAE2=ETA*ETA      S    B0E4=B0E2*B0E2
B1E7=B1E3*B1E4      S    B1E8=B1E4*B1E4
B2E4=B2E2*B2E2      S    DOE2=D0*D0
D1E2=D1*D1      S    D2E2=D2*D2
D1E2=D11*D11      S    D1E2=D12*D12
D13E2=D13*D13      S    D1E2=D14*D14
D16E2=D16*D16      S    D20E2=D20*D20
Y0E10=Y0E5*Y0E5      S    Y1E10=Y1E5*Y1E5
Y0C11=Y0C6/Y0      S    Y1C11=Y1C6/Y1
Y0C12=Y0C11/Y0      S    Y1C12=Y1C11/Y1
Y0C13=Y0C10/Y0      S    Y1C13=Y1C10/Y1
Y0C14=Y0C8/Y0      S    Y1C14=Y1C8/Y1
Y0C15=Y0*Y0C3      S    Y1C15=Y1*Y1C3
AA12010
AA12020
AA12030
AA12040
AA12050
AA12060
AA12070
AA12080
AA12090
AA12100
AA12110
AA12120
AA12130
AA12140
AA12150
AA12160
AA12170
AA12180
AA12190
AA12200
AA12210
AA12220
AA12230
AA12240
AA12250
AA12260
AA12270
AA12280
AA12290
AA12300
AA12310
AA12320
AA12330
AA12340
AA12350
AA12360
AA12370
AA12380
AA12390
AA12400
AA12410
AA12420
AA12430
AA12440
AA12450
AA12460
AA12470
AA12480
AA12490
AA12500

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TRANNOZ CODE... SUBROUTINE AATRANS (CONT.)

```

YOC16=YOC3*YOC11    S   YIC16=YIC3*YIC11
YOC17=YOE2*YOC8    S   YIC17=YIE2*YIC8
YOC18=YOE2*YOC9    S   YIC18=YIE2*YIC9
YOC19=YOC10*YOC0    S   YIC19=YIC10*YICO

C... CALCULATE THE "F" CONSTANTS:
C
F4=H1*(D13*YOE4/16.+D20*YOE2-BETA1*B1*Y0+B2*E2*YOC11+D14*YOC4*x.25
S+D16*YOC0+D0)          AAT2510
F5A=H2*ETAE2+H2*ETA*(B1E2*YOE2*x.5+B2*YOC0+B0)+H2*(D13*YOE4/16.+
SD20*YOE2-BETA1*B1*Y0+B2*E2*YOC11+D14*YOC4*x.25+D16*YOC0+D0)      AAT2520
F5B=H1*(D12*YOE2*x.2*D11*YOC0+D1)-GP1*x.5*H1*(0.75*B1E3*YOE2+
S6+B1*B2*YOC0+B1*B2-B3/YOE2)          AAT2530
F5=F5A+F5B          AAT2540
F6=H2*ETAE2*B1+H1*D2+H2*(D12*YOE2*x.2*D11*YOC0+D1)-GP1*x.5*H1*
S(B1E2-32/YOE2)-GP1*x.25*H2*(0.75*B1E3*YOE2+B6+B1*B2*YOC0+        AAT2550
SB1*B2-B3/YOE2)          AAT2560
F7=H2*D2-GP1*x.25*H2*(B1E2-B2/YOE2)          AAT2570
F8=G1*(D13*YIE4/16.+D20*YIE2-BETA1*B1*Y1+B2*E2*YIC11+D14*YIC4*x.25
S+D16*YICO+D0)          AAT2580
F9A=G2*ETAE2+G2*ETA*(B1E2*YIE2*x.5+B2*YICO+B0)+G1*(D12*YIE2*x.2*
SD11*YICO+D1)+G2*(D13*YIE4/16.+D20*YIE2-BETA1*B1*Y1+B2*E2*YIC11+
SD14*YIC4*x.25+D16*YICO+D0)          AAT2590
F9B=-GP1*x.5*G1*(0.75*B1E3*YIE2+B6+B1*B2*Y1C0+B1*B2-B3/YIE2)
F9=F9A+F9B          AAT2600
F10=G2*ETAE2*B1+G1*D2+G2*(D12*YIE2*x.2*D11*YICO+D1)-GP1*x.5*
SG1*(B1E2-B2/YIE2)-GP1*x.25*G2*(0.75*B1E3*YIE2+B6+B1*B2*YICO+
SB1*B2-B3/YIE2)          AAT2610
F11=G2*D2-GP1*x.25*G2*(B1E2-B2/YIE2)
F12=2.*GM1*B1*D12*x.2*GM1*B1E4*x.4.*D2*E2*x.4.*D2*x.D12*x.(G+2.)*
SB1E2*x.D2          AAT2620
F13=(F11*Y0-F7*Y1)*Y1*Y0/(YOE2-YIE2)
F14=F12*x.5+4.*B1*x.F3
F15=3.*B1E2*x.F3+(G+2.)*B1E3*x.D2*x.9.*B1*x.F14*x.6.*D2*x.D12*x.(G+2.)*
SB1E2*x.D12*x.2.*G*SB1E5+GM1*B1*x.D13          AAT2630
F16=GP1*x.5*BEТА1*x.B1E2-BETA1*x.D2
F17=(5.-G-1.)*.5*BEТА1*x.B2
F18=GP1*x.5*BEТА1*x.B2*x.D1
F19=6.*B2*x.F3*x.(G+2.)*B1*x.B2*x.D2*x.16.*B1*x.F13*x.12.*D2*x.D11*x.2.*{(2.*G+
S1.)*B1E2*x.D11*x.2.*GM1*x.B2*x.D12+GM1*x.B1*x.D14*x.3.*GM1*x.B1E3*x.B2
F20A=(3.*G+5.)*.5*SB1E3*x.B2*x.6.*B1E2*x.D11*x.6.*B2*x.D12*x.2.*GM1*x.B0*x.D12
S+2.*GM1*x.B1*x.D19+GM1*x.5*B1*x.D14*x.2.*GM1*x.B6*x.D2+GM1*x.B1E2*x
SB6          AAT2640
F20B=GM1*x.B1*x.B2*x.D2*x.(2.*G+1.)*B1E2*x.D1*x.2.*GM1*x.B0*x.B1E3*x.6.*B0*x.F3+
S6.*D1*x.D2*x.6.*B0*x.B1*x.D2          AAT2650
F20=F20A+F20B          AAT2660
F22=F15*YOE3*x.25+F16+F17*YOC5-F18/YOE2+F19*(YOC1*x.5-Y0*x.25)*F20*x
SY0*x.5          AAT2670
AAT2680          AAT2690

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TRANNOZ CODE... SUBROUTINE AATRANS (CONT.)

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F23=F15*YIE3*.25+F16+F17*YIC5-F18/YIE2+F19*(YIC1*.5-Y1*.25)+F20*
SY1=.5
F21=(F10*YO-F6*Y1-F23*Y0+F22*Y1)*Y1*YO/(YOE2-YIE2)
F2=(F6*YO-F21-F22*YO)/(3.*B1*YOE2)
F24=F20*.5-F19*.25+3.*B1*F2
F25=F24*.5-F19/.8
F26=3.*B1E2*F14+B1*F15*.5+.2.*D12E2+D2*D13*.25+(5.*G+4)/6.*B1E2*
S D13+(G+2.)*B1E3*D12+B1*F15*.5+(2.*G+1.)*5.*B1E6+G1*B1*D21
F27A=2.*B1E2*F2*GP1*B1E3*D1+2.*B0*B1E2*D2+.5*B0*F14+.8.*B1*F25+
S 4.*D1*D12+4.*B0*B1*D12+4.*D2*D20+.2.*G*B1E2*D20+.2.*G*B1E3*B6+(3.*G+7.)*.25*B1E4*B2
F27B=(G+.3.)*B1E2*D19+B2*D13+.2.*G*B0*B1E4+.2.*GP1*B6*D12
S+B1E3*D11+G1*.25*B1E2*D14+GM1*B0*D13+.4.*GM1*B1*
SD32+G1*.25*B1*D26+GM1*B1*B2*D12
F27=F27A+F27B
F28=.8.*B1*F16-.4.*BETA1*B1*D2-(19.*G+43.)/8.*BETA1*B1E3-.5.*BETA1*
S D12+GM1*B1*D23
F29=.8.*B1*F18-.2*(G+2.)*BETA1*B1*B2-.4.*BETA1*D11+GM1*BETA1*B0
S+4.*GP1*B1*D35-BET1*D1+GP1*.5*BET1*B6
F30=(3.*G-1.)*B1*B2*B3+.4.*B2*B0*B2E2+.4.*B3*D11
F31=GP1*.5*BETA1*B3
F32=.12.*B2*F13+.4.*B1*F17+.8.*B1E2+.6.*B2E2*D2+(3.*G-.3.)*B1E2*B2E2
S+4.*GP1*B1*B2*D011+GM1*B2*D14+GM1*B1*D25
F33=.6.*B1E2*F13+.6.*B2*F14+.2.*B1*F19+.8.*D11*D12*D2*D14+(2.*G+3.)*
S.5.*B1E2*D14+.2.*GP1*B1E3*B1*D11+.2.*G+3.)*B1*B2*D12+.2.*B1E2*B2*D2+
S 4.*G*B1E4*B2+GM1*B2*D13+GM1*B1*D26
F34A=.4.*B2*F2+.2.*GP1*B1*B2*D1+.4.*B0*B2*D2+.12.*B0*F13+.8.*B1*F21+
S 6.*D1*D11+.8.*B0*B1*D11+.4.*D2*D16+.2.*G*B1E2*D16+.2.**G+.6.)*B1E2*
SB2E2
F34B=(G+.3.)*.5.*B2*D14+.2.*GP1*B1*B2*D11+.2.*GM1*B2*D19+.GM1
S*B0*D14+GM1*B1*D25+.2.*GM1*B1*D34+.4.*GM1*B6*D11+.4.*GM1
S*B0*B1E2*B2+.2.*GM1*B1*B2*B6
F34=F34A+F34B
F35=GP1*.5*BETA1*B1*B2-.2.*BETA1*D11
F36=.2.*G+.4.)*B2E3
F37A=GP1*B1E3*B3+GP1*B1*B2*B6+.4.*B1E2*D16+.4.*B2*D19+G*B1E2*
S B2E2+(G+.3.)*B0*B1E2*B2+.4.*B3*D12+.4.*B6*D11+.2.*GM1*B0*D19+GM1
S.5*B0*D14
F37B=.2.*GM1*B0*B1*B6+GM1*B1*B2*D1+.2.*G*B1E2*D0+GM1*B0E2*B1E2*
S 4.*B0*F2+.2.*D1E2+.4.*D0*D2+.4.*B0*B1*D1+.2.*B0E2*D2
F37-F37A+F37B
F39=F26*YOE5/6.1+F27*YOE3*.25+F28*YOE2/3.+F29+F30*YOC5-F31/YOE2+
S Y32*(YOC6*.5.-YOC1*.5+YOC2*.25)+F33*(YOC7*.25-YOE3/16.)+F34*(YOC1*.5-
S YO*.25)+F35*(YOC0-1.1+F36*YOC12*.5+F37*Y0*.5
F40=F26*YIE5/6.1+F27*YIE3*.25+F28*YIE2/3.+F29+F30*YIC5-F31/YIE2+
S F32*(YIC6*.5-YIC1*.5+Y1*.25)+F33*(YIC7*.25-YIE3/16.)+F34*(YIC1*.5-
S Y1*.25)+F35*(YIC0-1.1+F36*YIC12*.5+F37*Y1*.5
F38=(F9*YO-F39*Y1-F40*Y0+F39*Y1)*Y1*YO/(YOE2-YIE2)
F1=(F5*YO-F39*Y0-F38)/(2.*B1*YOE2)

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TRANNOZ CODE... SUBROUTINE AATRANS (CONT.)

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F41=F27*.25-F33/16.
F42=F32*.25-F34*.25+F37*.5+2.*B1*B1
F43=F34*.5-F32*.5
F44=F29-F35
F45=F41*.25-F33/64.
F46=F42*.5+F32/8.-F43*.25
F47=F44-F35
F48=F43*.5-F32*.25
F49=B1E2*F15/8.+B1*F26/16.+D12*D13/8.+B1E4*D12*-25*(G+2.)/16.*B1E3
$*D13+(5.*G+3.)/32.*B1E7+(3.*G-1.)/6.*B1E2*D21
F50A=2.*B1E2*F25+B0*F15*.25+2.*B1!*F45+2.*D12*D20+D1*D13/8.+B0*B1*
$D13/8.+B0*B1E2*D12+B1E4*D1*.25+G*B1E3*D20+GP1*.5*B1E4*B6+2.*G*
$B1E2*D32
F50B=B2*D21/3.+GP1*/8.*B1E5*B2*G*.5*B0*B1E5+B1E3*D19*.5+
$(*G+3.)/8.*B6*D13+GM1/8.*B1E2*D26+GM1*B0*D21+GM1/16.*B1*
$B2*D13
F50=F50A+F50B
F51=2.*B1E2*F16+2./9.*B1!*F26-2.*BETA1*B1*D12-(2.*G+3.)*.5*BETA1*
$B1E4+(3.*G+1.)/6.*B1E2*D23-9./16.*BETA1*D13
F52A=B1E2*F1+B0*B1E2*D1+G*B1E3*D0+4.*B0*F25+2.*B1!*F46+2.*D0*D12+
$B0E2*D12+2.*D1*D20+2.*B0*B1*D20+(G+3.)*.25*B1E4*B3+GM1*.5*B1*
$B6E2
F52B=GP1*B1E2*D33+2.*B2*D32+B1E2*B2*B6+GP1*B0*B1E2*B6+G*
$5*B0*B1E3*B2+B1E3*D16*.5+2.*B6*D19+B3*D13*.5+GM1*.5*B1E2*
$D34
F52C=4.*GM1*B0*D32+GM1*.25*B0*D26+2.*GM1*B6*D20+GM1*
$B1*B2*D20+GM1*.5*B0E2*B1E3
F52=F52A+F52B+F52C
F53=2.*B1E2*F18+4.*B0*F16+2.*B1!*F47-2.*BETA1*B1*D1+(G-5.)*BETA1*
$B0*B1E2-(3.*G-1.)*.5*BETA1*B1*B6*(G+3.)*.5*B1E2*D35+2./3.*B2*D23-
$(3.*G-1.)*.5*BETA1*B1E2*B2-2.*BETA1*D19+GM1*B0*D23-BETA1*D20
F54=4.*B0*F18+2.*B1!*F31-GP1*BETA1*B1*B3+2.*B2*D35-GP1*
$BETA1*B0*B2-2.*BETA1*D16+GM1*B0*D35-BETA1*D0+GM1*BETA1*
$B0E2
F55=GM1*.5*B1*B3E2+2.*B2*D29+2.*B0*B0*B2*B3+2.*B2E2*D11+(3.*G-1.)*B1*B2E3+GM1*B2*
$F56=2.*B2*F17+B1*F36/3.+6.*B2E2*D11+(3.*G-1.)*B1*B2E3+GM1*B2*
$D25
F57=B1E2*F17+B2*F19+B1*F32*.5+3.*B2E2*D12+D1*D14+2.*B1E2*B2*D11+
$3.*G*B1E3*B2E2+(G+2.)*.5*B1*B2*D14+GP1*.5*B1E2*D25+GM1*B2*
$D26
F58=4.*B2*B21+2.*B0*F17+B1*F30+4.*D11*D16+3.*B2E2*D11+2.*G*B0*B1*
$B2E2*D11+2.*G*B1*B2*D25+3.*GM1*B2E2*B6
F59=-BETA1*B2E2
FC0=B1E2*F19*.5+B2*B21*.25+B1*F33/8.+D12*D14*.5*D11*D13*.25*B1E2*
$B2*D12+B1E4*D11*.5+GP1*.25*B1E3*D14+(G+4.)/8.*B1*B2*D13+(2.*G+
$1.)*.5*B1E5*B2*G*.5*B1E2*D26+GM1*B2*D21
F61A=2.*B1E2*F21+4.*B2*F25+B0*F19+2.*B1!*F48+2.*D12*D16+4.*D11*
$D20+D1*D14*.5*B0*B1*D14*.5+2.*B0*B1E2*D11+2.*B0*B2*D12+B1E2*B2*
$D1

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TRANNOZ CODE... SUBROUTINE AATRANS (CONT.)

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F61B=G*B1E3*D16+2.*G*B1*B2*D20+2.*G*B1E2*B2*B6+GP1*B1E2*D34+
SGP1*.25*B2*D26+(G+4.)*.5*B1E3*B2E2+2.*G*B0*B1E3*B2+GP1*.5*
SB6*D14 .25*B1*B2*D19+GM1*.5*B1E2*D25+4.*GM1*B2*D32+GM1*
F61C=2.*B1*B2*D19+GM1*.5*B1E2*D25+4.*GM1*B2*D32+GM1*
F61=F61A+F61B+F61C
F62=.25*BETA1*D14*GM1*B2*D23
S*B2-1.*B2*B1E6+2.*B1*B2*D16-4.*BETA1*B1*D11-(5.*G+11.)*.5*BETA1*B1E2
F63A=.2.*B2*B1E2+2.*B0*B2*D01+2.*G*B1*B2*D00+4.*B0*F21+2.*B1*F38+4.*
SD0*D11+2.*B0E2*D11+2.*D01*D16+2.*B0*B1*D16+(G+5.)*B1E2*B2*B3+
SGP1*B2*D34
F63B=6.*B2E2*B6+GP1*B0*B1*B2E2+GP1*B0*B2*D16+B3*D14
S+2.*GM1*B2*D33+GM1*B0*D25+2.*GM1*B0*D34+2.*GM1*
SB6*D16+GM1*B0E2*B1*B2+2.*GM1*B0*B2*B6
F63=F63A+F63B
F64=4.*B2*B1E8-(G+7.)*BETA1*B2E2+GM1*B2*D35+GM1*BETA1*BO*
SB2-BETA1*D16
F65=2.*(G+4.)*B2E2*B3
F66A=GM1*B1*B3*B6+(G+5.)*.5*BETA1E2*B1+2.*B1E2*D29+2.*B2*D33+
SG*B1E2*B2*B3+2.*B0*B1E2*B3+2.*B0*B2*B6+2.*B6*D16+2.*B3*D19+2.*
SGM1*B0*D33
F66B=GM1*B0*D34+2.*GM1*B6*D0+GM1*B0E2*B6+GM1*B1
S*B2*D0+GM1*.5*B0E2*B1*B2+2.*B0*B1+2.*D0*D1+B0E2*D1+2.*B0*
SB1*D0
F66=F66A+F66B
F68=F50/6.-F60/36.
F69=F52*.25+F57/32.-F61/16.
F70=F53/3.-F62/9.
F71=F58*.25-3./8.*F56-F63*.25+F66*.5
F72=F58*.5-0.75*F56
F73=F61*.25-F57/8.
F74=0.75*F56-F58*.5+F63*.5
F75=F64/2.*F59
F76=F54+2.*F59-F64
F77=F49*YOE/8.+F68*YOE4/5.+F69*YOE3+F70*YOE2+F71*Y0+
$F56*YOC13*.5+F57*YOC14*.25+F72*YOC6+F59*YOC11+F65*YOC12*.5+F60*
$YOC15/6.+F73*YOC7+F62*YOC4/3.+F74*YOC1+F75*YOC0+F55*YOC5+F76
F78=F49*YIE/8.+F68*YIE5+F51*YIE4/5.+F69*YIE3+F70*YIE2+F71*Y1+
$F56*YIC13*.5+F57*YIC14*.25+F72*YIC6+F59*YIC11+F65*YIC12*.5+F60*
$YIC15/6.+F73*YIC7+F62*YIC4/3.+F74*YIC1+F75*YIC0+F55*YIC5+F76
F67=(F8*YO-F4*YI-F78*YO+F77*YI)*Y1*YO/(YOE2-YIE2)
FO=(F4*YO-F77*YO-F67)/(B1*YOE2)
F79=F71+B1*FO
C... CALCULATE THE Y CONSTANTS AND THE "E" FUNCTIONS:
C 70 IF(FCODEF) GO TO 80
      YE6=YE3*YE3      S   YE7*YE3*YE4
      YC8=YC4/Y      S   YC9=YC4*YC0
      YC10=Y*YC6     S   YC11=Y*YC7
      AT4000
      AT4010
      AT4020
      AT4030
      AT4040
      AT4050
      AT4060
      AT4070
      AT4080
      AT4090
      AT4100
      AT4110
      AT4120
      AT4130
      AT4140
      AT4150
      AT4160
      AT4170
      AT4180
      AT4190
      AT4200
      AT4210
      AT4220
      AT4230
      AT4240
      AT4250
      AT4260
      AT4270
      AT4280
      AT4290
      AT4300
      AT4310
      AT4320
      AT4330
      AT4340
      AT4350
      AT4360
      AT4370
      AT4380
      AT4390
      AT4400
      AT4410
      AT4420
      AT4430
      AT4440
      AT4450
      AT4460
      AT4470
      AT4480
      AT4490

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TRANS2 CODE... SUBROUTINE TRANS2 (CONT.)

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Y12=Y*YC9      S   YC13=Y*YC10
YC14=Y*YC11
E3P=F14*xY+F13/Y
E3=F13*YE2*a. 5+F13*YC0
E2P=F15*YE3*a. 25+F24*YC0
E2=F15*YE4/16. +F23*YE2+F16*Y+F15*Y+F17*TC48. 5+F18*TC50. 25+F21*TC50
E1P=F26*YE5/6. +F41*YE3+F28*YE2/3. +F42*Y+F38*Y-F31/Y+TC2+F39*TC3. 5
S+F38*YC08. 5+F39*YC7. 25+F43*YC1+F35*YC0+F30*YC9+F44
E1=F28*YE6/36. +F45*TE2+F47*Y+F31/Y+F38*YC8. 5
S+F32*YC10. 25+F30*YC4. S(Y*YC30*YC1/16. +F48*YC0+F34*YC1+F28*YC0
E0P=F49*YE7/6. +F68*YE8+F51*YE4/5. +F63*TE2+F70*TE2+F78*Y+F87/Y+
S+F56*YC12. 5+F57*YC13. 25+F72*YC3+F58*YC4+F59*YC3. 5+F80*YC14/Y+
S+F73*YC7+F62*YC5/3. +F74*YC1+F78*YC0+F38*YC9+F76

C... CALCULATE THE DESIRED QUANTITIES TO THIRD ORDER:
C
C   ZE3=Z*ZE2      S   EP3E3=EP2E3*EP2E2
U3=E1+F0*(2.*ZE2+F1)*Z+(3.*EP3+EP2)*Z*ZE2+F3*ZE3
V3=EP+EP*Z+EP*ZE2+EP*ZE3
U3U+U3*EP*ZE3      S   V=V*CYT*NUCE3*ZE3
NSTAR=PI*STAR+(U3+EP1*a. 25*V1*V1)*EP2E3
THETA=THETA+CYT*(V3-U1*V2-U2*V1+V1*U1*V1)*EP2E3
R=PI+CHI*(U3+EP1*a. 25*V1*V1+1. S*EP1*NU1*U1*V1)+EP2E3
S6. S(U1*U1*U1)*EP3E3
PP0=PP0-CPP0*G0*(U3+EP1*a. 25*V1*V1-EP1/a. 25*NU1*U1)
U3S=S*NL(U3)      S   V3=S*NL(V3)
US=S*NL(U)        S   VS=S*NL(V)
NSTARS=SNGL(NSTAR)      S   THETAS=SNGL(THETA)
HS=SNGL(H)        S   PPOS=SNGL(PP0)
RETURN

C... IF DESIRED, CALCULATE THE DISCHARGE COEFFICIENT:
C
80   C036=B1*E3*D32*a. 25+B6*D21/6.
C037=B1*E3*D23/12. -BETA1*D21/6.
C038=B1*E3*D33*a. 25+B6*D22+93*a. 021/6.
C039=B1*E3*D35*a. 25+B6*D23/3. -BETA1*D22
C040=B1*E3*D29*a. 25+B6*D33+B3*D32-BETA1*D23/3.
C041=B6*D35+B3*D23/3. -BETA1*D23
C042=B6*D29+B3*D33-BETA1*D35
C043=B3*D35-BETA1*D29
C044=B1*E3*D25/a. 5+B1*B2*D26. 25
C045=B6*D23*a. 5+B1*B2*D26. 25
C046=-BETA1*B1*B2*E2-BETA1*D25*a. 5
C047=2.*B1*B2*E2*B3*D3*D26*a. 5
C048=B1*E3*D26/16. +B1*B2*D21/6.
C049=B1*E3*D34*a. 25+B6*D22*a. 25+B1*B2*D32
C050=B1*B2*D23/3. -BETA1*B1*E3*D22*a. 25-BETA1*D26*a. 25
C051=B1*E3*B2*B3*a. 5+B6*D34+B1*B2*D33+B3*D26*a. 25
C052=-BETA1*D22*B6+B1*B2*D35-BETA1*D34

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TRANNOZ CODE . . SUBROUTINE AATRANS (CONT.)

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CD53=2.*B2*B3*B6+B1*B2*D29+B3*D34+BETA1E2*B2
CD54=CD36/8.-CD48/64.
CD55=CD38/6.+CD44/108.-CD49/36.
CD56=CD39/5.-CD50/25.
CD57=CD40*/25.-256.*B1*B2*D25+CD45/32.-CD51/16.
CD58=CD41/3.*2./27.*CD46-CD52/9.
CD59=CD42*.5+CD47*.25-CD53*.25
CD60=CD43*.3.*BETA1*B2*B3
CD61=CD45*.25-3./32.*B1*B2*D25
CD62=CD49/6.-CD44/18.
CD63=.3./64.*B1*B2*D25-CD45/8.+CD51*.25
CD64=CD52/3.-2./9.*CD46
CD65=CD53*.5-CD47*.5
CD30A=B1E3*D21/240.*((YOE10-YIE10)+CD54*(YOE8-YIE8)+CD37/7.*((YOE7-
3*YIE7)+CD55*(YOE6-YIE6)+CD56*(YOE5-YIE5)+CD57*(YOE4-YIE4)+CD58*-
3(YOE3-YIE3))
CD30B=CD59*(YOE2-YIE2)+CD60*(Y0-Y1)+B1*B2*D25/8.*((YOC16-
3YIC16)+CD44/6.*((YAC17-YIC17)+CD61*(YOC8-YIC8)+CD46/3.*((YOC14-
3YIC14))
CD30C=CD47*.5*((YOC2-YIC2)+B2*B3E2*((YOC11-YIC11)+CD48/8.*((YOC16-
3YIC16)+CD62*(YOC9-YIC9)+CD50/5.*((YOC15-YIC15)+CD63*(YOC3-YIC3)+-
3CD64*(YOC7-YIC7)
CD30D=CD65*(YOC4-YIC4)-3.*BETA1*B2*B3*((YOC1-YIC1)+B3*D29*-
3(YOC0-YIC0))
CD30=CD30A+CD30B+CD30C+CD30D
CD66=B1E5*B6*.25+BO*B1E6/16.
CD67=B1E5*B3*.25+B1E2*B6E2*.5+BO*B1E3*B6*.5
CD68=-BETA1*B1E2*B6-BETA1*B0*B1E3*.5
CD69=B1E2*B3*B6+BETA1E2*B1E2*.5+BO*B1E3*B3*.5+BO*B6E2
CD70=-BETA1*B1E2*B3-2.*BETA1*B0*B6
CD71=2.*B0*B3*B6+BETA1E2*B0+B1E2*B3E2*.5
CD72=2.*B1*B2E2*B6+BO*B1E2*B2E2
CD73=1.5*B1E3*B2*B6+B0*B1E4*B2*.5
CD74=1.5*B1E3*B2*B3+B2*B6E2+2.*BO*B1*B2*B6
CD75=-2.*BETA1*B2*B6-2.*BETA1*B0*B1*B2
CD76=2.*B2*B3*B6+BETA1E2*B2+2.*BO*B1*B2*B3
CD77=CD66/.8.-5./1024.*B1E6*B2
CD78=CD67/.6.+B1E4*B2E2/108.-CD73/36.
CD79=CD68/.5.*3./50.*BETA1*B1E3*B2
CD80=CD69*.25-3./128.*B1E2*B2E3+CD72/32.-CD74/16.
CD81=CD70/.3.-4./27.*BETA1*B1*B2E2-CD75/9.
CD82=CD71*.5+B1*B2E2*B3*.5-CD76*.25
CD83=2.*BETA1*B2*B3-2.*BETA1*B0*B3
CD84=CD72*.25-3./16.*B1E2*B2E3
CD85=CD73/.6.-B1E4*B2E2/18.
CD86=.3./32.*B1E2*B2E3-CD72/8.+CD74*.25
CD87=4./9.*BETA1*B1*B2E2+CD75/3.
CD88=CD76*.5-B1*B2E2*B3
CD31A=B1E8/320.*((YOE10-YIE10)+CD77*(YOE8-YIE8)-BETA1*B1E5/-
328.*((YOE7-YIE7)+CD78*(YOE6-YIE6)+CD79*(YOE3-YIE3)+CD80*-
3(YOE4-YIE4))

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TRANNOZ CODE... SUBROUTINE AATRANS (CONT.)

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CD31B=CD81*(Y0E3-Y1E3)+CD82*(Y0E2-Y1E2)+CD83*(Y0-Y1)+81E2*          AAT5510
SB2E3=.25*(Y0C16-Y1C16)+B1E4*B2E2/6.*(Y0C17-Y1C17)+CD84*(Y0C8-Y1C8)    AAT5520
CD31C=-2./3.*BETA1*B1*B2E2*(Y0C14-Y1C14)+B1*B2E2*B3*(Y0C2-Y1C2)+B2    AAT5530
$*B3E2*.5*(Y0C11-Y1C11)+5./120.*B1E6*B2*(Y0C16-Y1C16)+CD85*(Y0C9-    AAT5540
SY1C9)
CD31D=-3./10.*BETA1*B1E3*B2*(Y0C15-Y1C15)+CD86*(Y0C3-Y1C3)+CD87*    AAT5550
$(Y0C7-Y1C7)+CD88*(Y0C4-Y1C4)-2.*BETA1*B2*B3*(Y0C1-Y1C1)+B0*B3E2*    AAT5560
$(Y0C0-Y1C0)
CD31=G01*5*(CD31A+CD31B+CD31C+CD31D)                                AAT5580
CD89=B1E2*F45*.5+B0*F26/36.                                              AAT5590
CD90=B1E2*F46*.5+B0*F45                                              AAT5600
CD91=B1E2*F47*.5+B0*F28/9.                                              AAT5610
CD92=B1E2*F0*.5+B0*F46                                              AAT5620
CD93=B1E2*F31*.5+B0*F47                                              AAT5630
CD94=B1E2*F36/12.+B2*F32*.25.                                            AAT5640
CD95=B2*F30*.5+B0*F36/6.                                                 AAT5650
CD96=B1E2*F32/.8.+B2*F33/16.                                             AAT5660
CD97=B1E2*F30*.25+B2*F48+B0*F32*.25.                                    AAT5670
CD98=B2*F38+B0*F30*.5                                                 AAT5680
CD99=B1E2*F33/32.+B2*F26/36.                                              AAT5690
CD100=B1E2*F48*.5+B2*F45+B0*F33/16.                                         AAT5700
CD101=B1E2*F35*.5+B2*F28/9.                                              AAT5710
CD102=B1E2*F36*.5+B2*F46+B0*F46                                         AAT5720
CD103=B2*F47+B0*F35                                              AAT5730
CD104=B2*F0+B0*F38                                              AAT5740
CD105=CD89/8.-CD99/64.                                                 AAT5750
CD106=CD90/6.+CD96/108.-CD100/36.                                         AAT5760
CD107=CD91/5.-CD101/25.                                              AAT5770
CD108=CD92*.25+CD97/32.-CD102/16.-3./128.*CD94                         AAT5780
CD109=CD93/3.+2./27.*B2*F35-CD103/9.                                         AAT5790
CD110=B0*F0*.5+B2*F36/8.-3./8.*CD95+CD96*.25-CD104*.25               AAT5800
CD111=B0*F31-B2*F31                                              AAT5810
CD112=CD95*.5-B2*F36/6.                                                 AAT5820
CD113=CD97*.25-3./16.*CD94                                              AAT5830
CD114=B2*F36*.25-0.75*CD95+CD96*.5.                                         AAT5840
CD115=CD100/6.-CD96/18.                                                 AAT5850
CD116=CD102*.25-CD97/8.+3./32.*CD94                                         AAT5860
CD117=CD103/3.-2./9.*B2*F35                                              AAT5870
CD118=0.75*CD95-B2*F36*.25-CD96*.5+CD104*.5.                            AAT5880
CD32A=B1E2*F26/720.*$(Y0E10-Y1E10)+CD105*(Y0E8-Y1E8)+B1E2*F28/    AAT5890
$126.*$(Y0E7-Y1E7)+CD106*(Y0E6-Y1E6)+CD107*(Y0E5-Y1E5)+CD108*(Y0E4    AAT5900
$-Y1E4)
CD32B=CD109*(Y0E3-Y1E3)+CD110*(Y0E2-Y1E2)+CD111*(Y0-Y1)+B2*F36/    AAT5910
$12.*$(Y0C19-Y1C19)+CD94*.25*(Y0C16-Y1C16)+CD96/6.*$(Y0C17-Y1C17)+    AAT5920
$CD113*(Y0C8-Y1C8)                                                       AAT5930
CD32C=B2*F35/3.*$(Y0C14-Y1C14)+CD114*(Y0C2-Y1C2)+CD99/8.*    AAT5940
$(Y0C18-Y1C18)+CD115*(Y0C9-Y1C9)+CD101/5.*$(Y0C15-Y1C15)+CD116    AAT5950
$*(Y0C3-Y1C3)                                                       AAT5960
CD32D=CD117*(Y0C7-Y1C7)+CD118*(Y0C4-Y1C4)+B2*F31*(Y0C1-Y1C1)    AAT5970
$+CD112*(Y0C10-Y1C10)                                              AAT5980
AAT5990
AAT6000

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TRANNOZ CODE... SUBROUTINE AATRANS (CONT.)

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CD32=2.* (CD32A+CD32B+CD32C+CD32D)
CD119=D0*D13/8.+D20E2
CD120=2.*D0*D20+BETA1E2*B1E2
CD121=B2E2*D13/8.+D14E2/16.
CD122=2.*B2E2*D20+D14*D16*.5
CD123=2.*B2E2*D20+D16E2
CD124=013*D16/8.+D14*D20*.5
CD125=2.*D16*D20+D0*D14*.5
CD126=013*D20/64.-D13*D14/2048.
CD127=CD119/6.+CD121/108.-CD124/36.
CD128=BETA1*B1*D14*D10.-2./5.*BETA1*B1*D20
CD129=CD120*25-3./256.*B2E2*D14+CD122/32.-CD125/16.
CD130=2./9.*BETA1*B1*D16-2./3.*BETA1*B1*D0-4./27.*BETA1*B1*B2E2
CD131=00E2*.5+0.75*B2E2*D16+CD123*.25-D0*D16*.5
CD132=B2E2*D16-B2E4
CD133=CD122*.25-3./32.*B2E2*D14
CD134=1.5*B2E4-1.5*B2E2*D16+CD123*.5
CD135=CD124/6.-CD121/18.
CD136=3./64.*B2E2*D14-CD122/8.+CD125*.25
CD137=4./9.*BETA1*B1*B2E2-2./3.*BETA1*B1*D16
CD138=1.5*B2E2*D16-1.5*B2E4-CD123*.5+D0*D16
CD32A=013E2/2560.*(Y0E10-Y1E10)+CD126*(Y0E8-Y1E8)-BETA1*B1*D13/56.
$*(Y0E7-Y1E7)+CD127*(Y0E6-Y1E6)+CD128*(Y0E5-Y1E5)+CD129*(Y0E4-Y1E4)
$+CD130*(Y0E3-Y1E3)
CD32B=CD131*(Y0E2-Y1E2)+B2E4*.5*(Y0C19-Y1C19)+B2E2*D14/8.
$*(Y0C16-Y1C16)+CD132*(Y0C10-Y1C10)+CD121/6.*(Y0C17-Y1C17)+
$CD133*(Y0C8-Y1C8)
CD32C=-2./3.*BETA1*B1*B2E2*(Y0C14-Y1C14)+CD134*(Y0C2-Y1C2)
$+D13*D14/256.*(Y0C18-Y1C18)+CD135*(Y0C9-Y1C9)-BETA1*B1*D14/
$10.*(Y0C15-Y1C15)
CD32D=CD136*(Y0C3-Y1C3)+CD137*(Y0C7-Y1C7)+CD138*(Y0C4-Y1C4)
CD32A=CD33A+CD33B+CD33C+CD33D
CD139=B1E4*D20*.25+B0*B1E2*D13/16.
CD140=B1E4*D0*.25+B0*B1E2*D20+B0E2*D13/16.
CD141=B0*B1E2*D0+B0E2*D20
CD142=B1E2*B2E3+B2E2*D14*.25
CD143=B2E2*D16*.25+B0*B2E3
CD144=B1E4*B2E2*.25+B1E2*B2*D14*.25+B2E2*D13/16.
CD145=B1E2*B2*D16+B2E2*D20+B0*B1E2*B2E2+B0*B2*D14*.5
CD146=B2E2*D0*.25+B1E2*B2*D20+B0*B1E2*D16+B0E2*B2E2
CD147=B1E4*D14/16.*B1E2*B2*D13/16.
CD148=B1E4*D16*.25+B1E2*B2*D20+B0*B1E2*D14*.25+B0*B2*D13/8.
CD149=B1E2*B2*D0*B0*B1E2*D16+.25*B0*B2*D20+B0E2*D14*.25
CD150=2.*B0*B2*D0+B0E2*D16
CD151=CD139/8.-CD147/64.
CD152=CD140/6.+CD144/108.-CD148/36.
CD153=BETA1*B1E3*B2/25.-BETA1*B0*B1E3*.5.
CD154=CD141*.25-3./128.*CD142+CD145/32.-CD149/16.
CD155=2./9.*BETA1*B0*B1*B2-BETA1*B0E2*B1/3.-2./27.*BETA1*B1*B2E2
CD156=B0E2*D0*.5+0.75*B2E4-3./6.*CD143+CD146*.25-C0150*.25

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## TRANNSQZ C5DE... SUBROUTINE AATRANS (CONT.)

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CD157=CD143*.5-B2E4
CD158=CD145*.25-3./16.*CD142
CD159=1.5*B2E4-0.75*CD143+CD146*.5
CD160=CD148/6.-CD144/18.
CD161=.3/.32.*CD142-CD145*.8.+CD149*.25
CD162=.2/.9.*BETA1*B1*B2E2-2./3.*BETA1*B0*B1*B2
CD163=0.75*CD143-1.5*B2E4-CD146*.5+CD150*.5
CD34A=B1E4*D13/640.*(YOE10-YIE10)+CD151*(YOE8-YIE8)-BETA1
3*B1E5/28.* (YOE7-YIE7)+CD152*(YOE6-YIE6)+CD153*(YOE5-YIE5)+
SCD154*(YOE4-YIE4)
CD34B=CD155*(YOE3-YIE3)+CD156*(YOE2-YIE2)+B2E4*.5*(YOC19-
3-YIC19)+CD142*.25*(YOC16-YIC16)+CD157*(YOC10-YIC10)+CD144/6.
3*(YOC17-YIC17)
CD34C=CD158*(YOC8-YIC8)-BETA1*B1*B2E2/3.* (YOC14-YIC14) +
SCD159*(YOC2-YIC2)+CD147/6.* (YOC18-YIC18)+CD160*(YOC9-YIC9)-BETA1
3*B1E3*B2/5.* (YOC15-YIC15)
CD34D=CD161*(YOC3-YIC3)+CD162*(YOC7-YIC7)+CD163*(YOC4-YIC4)
CD34=(2.*G-3.)*(CD34A+CD34B+CD34C+CD34D)
CD164=B0*B1E6/16.-B1E6*B2/128.
CD165=B0E2*B1E4*.25+B1E4*B2E2/72.-B0*B1E4*B2/12.
CD166=B0E3*B1E2*.5-3./64.*B1E2*B2E3+3./16.*B0*B1E2*B2E2-3./6.*:
B0E2*B1E2*B2
CD167=B0E4*.5+0.75*B2E4-1.5*B0*B2E3+1.5*B0E2*B2E2-B0E3*B2
CD168=2.*B0*B2E3-B2E4
CD169=1.5*B0*B1E2*B2E2-3./6.*B1E2*B2E3
CD170=1.5*B2E4-3.*B0*B2E3+3.*B0E2*B2E2
CD171=B0*B1E4*B2*.5-B1E4*B2E2/12.
CD172=.16.*B1E2*B2E3-0.75*B0*B1E2*B2E2+1.5*B0E2*B1E2*B2
CD173=3.*B0*B2E3-1.5*B2E4-3.*B0E2*B2E2+2.*B0E3*B2
CD35A=B1E8/160.* (YOE10-YIE10)+CD164*(YOE8-YIE8)+CD165
3*(YOE6-YIE6)+CD166*(YOE4-YIE4)+CD167*(YOE2-YIE2)+B2E4*.5
3*(YOC19-YIC19)
CD35B=B1E2*B2E3*.5*(YOC16-YIC16)+CD168*(YOC10-YIC10)+B1E4
3*B2E2*.25*(YOC17-YIC17)+CD169*(YOC8-YIC8)+CD170*(YOC2-YIC2) +
3*B1E6*B2/16.* (YOC18-YIC18)
CD35C=CD171*(YOC9-YIC9)+CD172*(YOC3-YIC3)+CD173*(YOC4-YIC4)
CD35=(2.*G*G-5.*G+2.)*.25*(CD35A+CD35B+CD35C)
CD3=CD30+CD31+CD32+CD33+CD34+CD35
CD=CD-CCD*CD3*EPS*EPS
CDS=SNGL(CD)
RETURN
END

```